Transducer Design for Catapult Application
MCE 313 - Experiment 4

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Abstract

The objective of this lab is to develop a tool to calculate the total distance a ball travels when launched by a catapult. This is critical for medieval siege warfare. The goal is reached by using a combination of two major tools: strain indicator and full Wheatstone Bridge. By comparing the known distance that the ball travels with the distance the ball travels outputted by the strain indicator, the accuracy of the system can be determined. The system for this experiment is a catapult with a 1095 steel cantilever beam that acts as a spring to launch the marble out of the catapult. A Wheatstone Bridge is capable of measuring a variety of properties such as force, pressure, displacement, temperature, vibration, velocity, acceleration and many other properties [1, 4]. For the purpose of this experiment a full Wheatstone Bridge was used in conjunction with four strain gauges to the bottom of the beam. The strain indicator reading micro-strain utilizes an amplification factor applied to represent actual horizontal distance. An energy balance equation was determined representing the system, equating elastic and potential energy to kinetic energy. This equation determines the amplification factor at various strain values. This experiment was conducted at two strain values each of which had five trials. The different distances were calculated by a difference in strain between the initial and final position. The strain values used are: 3000 and 4000 micro-strain. For each trial the mean and standard deviation are calculated between the strain indicator projected value and the known distance traveled. The distance traveled is known by taking a video recording with a reference ruler. The mean and standard deviation for the difference of the values for the 3000 micro-strain trials is 0.4467 inches and 0.35114 inches, respectively. The mean and standard deviation for the difference in values for the 4000 micro-strain trials is 0.945 inches and 0.59083 inches, respectively. These results alone do not represent the strain indicator system being accurate. The standard deviations for both trials are a high percentage of the average value. This is not good as the standard deviation is the average distance of a data point from the average, meaning that the variance between the values is quite large.

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Nomenclature

Symbols & Units

g	Gravity
h	Height
m	Mass
r	Radius
v	Velocity
$\not{V}$	Volume
A	Amplification factor
dR/R	Change in resistance due to strain
E	Young's Modulus
E_i	Voltage supplied by the circuit
I	Moment of inertia
I_{cm}	Inertia about a center of mass
KE	Kinetic energy
PE	Potential energy
S_g	Strain gauge factor
V_o	Voltage measured across the Wheatstone bridge
ϵ	Strain
ω	Angular velocity
σ	Stress

1 Introduction

The objective of this experiment is to strategically incorporate a Wheatstone Bridge to determine the total distance a projectile will travel when launched by a Da Vinci Catapult. This will allow for the strategic placement of this siege weapon. This tool is valuable for a variety of reasons. The initial purpose of this tool is to determine the distance in which the catapult is to be placed. A catapult is to be close enough in order to strike its target. However, placing the catapult too close leaves catapult operators vulnerable to defending archers. The second purpose will be to improve accuracy. It is very common for catapult projectiles to have irregular mass. This tool will allow a user to select projectiles more strategically as well as allow the user to understand more where the projectile will strike. [2]

A Wheatstone Bridge is a widely applicable tool that is capable of measuring several properties including force, pressure, displacement, temperature, vibration, velocity, acceleration, and many more. This experiment uses strain gauges, placed in a specific orientation, to measure strain. The bridge is a parallel electric circuit where each leg has two resistors. When using a full bridge, each resistor is represented by a device that measures one of the properties mentioned above. In this case, the change in resistance is due to the deformation of strain gauges. Strain gauges are devices that measure the mechanical strain of an object which, using a Wheatstone bridge, is measured as electrical resistance. The strain gauge in cooperation with the Wheatstone Bridge, allows an output of strain which can be related to other properties. A type II full Wheatstone bridge is used in order to calculate strain within the leaf spring of the Da Vinci Catapult. A type II full Wheatstone Bridge utilizes four strain gauges to calculate strain due to bending [1, 2, 4, 5, 9].

The Da Vinci catapult came as a kit, a beam of 1095 steel was used in lieu of the bamboo spring which acts as the load to launch the catapult. The main goal is to calculate the distance the ball travels; and have that value appear on the strain indicator when the catapult is fully deflected and ready to launch. This method is based on energy balance equations involving potential energy, rotational energy, strain energy, elastic energy and kinetic energy.

For this lab, the output of strain, using the Wheatstone Bridge, is used to determine the distance a known projectile is launched. These calculations will be related to the actual distance the projectile travels. To calculate the distance of the projectile, the transducer is used to determine strain. Strain is necessary as it is used to calculate strain energy, a potential energy. This equation follows Hooke's law for continuous media. The strain will be read at two deflection locations: initial strain felt when the catapult is loaded at maximum, and the deflection of the beam after launch. These measurements are taken in reference to the unloaded position, where the beam is under no strain. Due to the characteristics of the spring, the correlation of strain and deflection is linear [3, 6, 7].

When all of the values required to compute the velocity of the ball as it leaves the basket, the distance the ball traveled can be determined. The distance can be verified based on a video taken of the catapult being launched with a yard stick in view as a reference. This value is used to calculate an amplification factor. The amplification factor is used to alter the strain indicator so that micro-strain is read as horizontal ball traveled. Different amplification factors are necessary when the initial strain deflection is altered.

The results of this lab is to determine how accurate the strain indicator can be at reading the distance the ball traveled in terms of inches. The values recorded from the video will be statistically analyzed and compared to the strain indicator values. The mean and standard deviation of the difference of the data sets for $3000\mu\epsilon$ is: 0.4467 inches with a standard deviation of 0.35114 inches The mean and standard deviation for the $4000\mu\epsilon$ trial is 0.945 inches with variance of 0.59083 inches. Based on the standard deviations in both trials being so high in relation to the mean, the strain indicator is not accurate in its readings of the distance that the ball traveled. A typical data points average distance from the mean is highly proportional to the mean itself. This suggests much variance in the data collected.

2 Theory

The objective of this experiment is to establish a relationship with the energy contained within the spring and the range of a catapult. This requires an understanding of the material mechanics of the spring, the energy balance model of the catapult and the kinematics of the projectile. The focus of this lab is to measure and understand the material mechanics of the spring system by means of a transducer. In order to understand the spring strain to force relationship, a transducer is used to measure the change in strain. This strain is then used to represent the potential energy of the spring. Using the potential strain energy to represent the spring in the energy balance model of the catapult the throwing velocity of the arm is determined. Using the velocity of the arm and the release angle, kinematics is then used to establish range.

2.1 Mechanics of the Spring

Computing strain energy of the spring begins with an understanding of strain and how strain is measured using a transducer. The transducer receives input as resistance which is then measured as an electrical system. In this case, the resistance is measured by strain gauges. The configuration of the resistors is known as a Wheatstone bridge. A primary purpose of a Wheatstone bridge is to measure unknown resistances and to calibrate measuring instruments. Other tools can be easily used to measure the resistance, however, the Wheatstone bridge is unique in that it can measure extremely low resistance values, including milli-Ohms. The Wheatstone bridge is a circuit comprised of two series- parallel resistance arrangements. These arrangements are connected between a voltage supply terminal and the ground, which ideally produces no voltage difference between the two parallel branches. The bridge has two input and two output terminals, with a total of four resistors. The resistance is measured between resistors across both legs of the series circuit. When all four resistors are represented by a sensor this is considered a full Wheatstone Bridge. A full Wheatstone bridge is used in this experiment configured to read strain.

A full Wheatstone bridge that measures bending strain is very sensitive to strain while rejecting axial strain. It is able to account for lead resistance and temperature fluctuation. This is possible when the strain gauges are configured in an orientation which measures bending deflection. These strain gages are connected to the transducer containing a Wheatstone bridge. The fundamental strain gauge equation is as follows:

$$\frac{dR}{R} = S_g \epsilon \tag{1}$$

$\frac{dR}{R}$ represents the change in resistance due to strain. This is represented by the strain , ϵ, times the strain gauge factor S_g.

The transistor relates the change in resistance due to strain by reading the change in voltage across the bridge. This is understood by the following equation:

$$V_o = \frac{-AE_i}{4} S_g(|\epsilon_1| + |\epsilon_2| + |\epsilon_3| + |\epsilon_4|) \tag{2}$$

Voltage supplied to the circuit and voltage measured across the Wheatstone bridge, represented by E_i and V_o respectively, are measured in millivolts. A is the amplification factor. V_o represents strain. For this experiment the excitation voltage is the voltage supplied.

The strain gauges themselves are only accurate with in a cretin range. This range is dictated, in this experiment the range is dictated by the end of the capabilities and the point at which the beam leaves it's the elastic portion. The strain equation is as follows:

$$s = \frac{E_i k R}{E_i} \tag{3}$$

Due to the use of predetermined strain values, our range is negligible. For application, range and scale of a full scale model should be evaluated.

The strain read by the transducer is related to the potential energy within the spring by the elastic strain energy equation:

$$PE = \frac{1}{2} V\!\!\!/ \sigma\epsilon = \frac{1}{2}\frac{V\!\!\!/}{E}\sigma^2 = \frac{1}{2} V\!\!\!/ E\epsilon^2 \tag{4}$$

The above equation includes volume $V\!\!\!/$, Young's Modulus E, strain ϵ, and stress σ.

Stress is the deformation of a part due to an applied load. Stress follows Hooke's Law for continuous media.

$$\sigma = \epsilon E \tag{5}$$

where σ is the bending stress, ϵ is the strain of the beam measured by the transducer, and E is the Young's modulus.

The potential energy within the spring is considered in the energy balance system seen in section 2.2.

2.2 Energy Balance of a Da Vinci Catapult

The distance that a projectile is launched is directly related to the energy and direction of energy applied to the projectile. The primary energy or load for a Da Vinci catapult is contained within the beam/spring, see section 2.1. Energy balance follows the Conservation of Energy which states that "Energy cannot be created nor destroyed". As a result the equations is as follows:

$$KE_i + PE_i = KE_o + PE_o \tag{6}$$

This is understood as: the kinetic energy, KE and potential energy, PE into a system is equal to the kinetic and potential energy out of the system. Kinetic energy is the the energy an object has due to motion. Potential energy is the energy possessed by and object because of its position relative to another object, stresses within itself, its electric chart, or other factors.

In this experiment there are three components that are in motion; the spring (Denoted as $_s$), the arm of the catapult (Denoted as $_a$), and the projectile/ball (Denoted as $_b$). The initial position of the catapult is when the arm is cocked, the spring is at its maximum deflection, and the projectile sits in the cup of the arm. The final position of the catapult is the moment before the arm strikes the stopper, the ball is leaving the cup of the arm and the spring has retracted to its launched position. It is important to consider the arm, a moment before it stops, in order to account for energy lost due to the inertia of the arm. The spring is not directly attached to the arm, rather by a string, and therefore does not factor into the velocity of the ball at the moment of its release. Each component is represented by both a potential and a kinetic energy as stated above. The spring is assumed to not be in motion in both the initial and final stages of its energy transfer. Therefore, the kinetic energy of the spring is neglected. The Potential energy of the arm and the spring are as follows:

$$PE = mgh \tag{7}$$

where m is the mass felt by the system, g is gravity, and h is the height relative to the datum. For this experiment the datum is the ground. The ground is at constant altitude representing flat terrain.

The potential energy of the spring considered by equation (3) found in section 2.1, is determined using strain.

The kinetic energy of the ball and arm is in the form of rotational kinetic energy. For this experiment, motion is only found after the catapult has been fired. Therefore, kinetic energy is only present on the right hand portion of the energy balance. The equation for rotational kinetic energy is as follows:

$$KE = \frac{1}{2}I\omega^2 \tag{8}$$

This is understood as: The kinetic energy, KE is equal to one half the moment of inertia, I times the angular velocity, ω squared. Our goal is to determine the distance the ball travels due to its velocity. In order to accomplish this the angular velocity is related to linear velocity using the following equation:

$$\omega = \frac{v}{r} \tag{9}$$

Where the angular velocity, ω is equal to the velocity, v of the ball over the radius, r measured from the axis of rotation to the center of mass.

The moment of inertia is tensor that describes the torque required to rotate an object at a desired angular acceleration about a rotational axis. The moment of inertia assumes the center of mass is in line with the axis of rotation. Neither the ball nor the arm have their center of masses in line with the axis of rotation. As a result we need to determine the second moment of inertia as described by the parallel axis theorem:

$$I = I_{cm} + mr^2 \tag{10}$$

Where I_{cm} is the inertia about the center of mass at a distance, r^2 times the mass, m of the object.

Using this the kinetic energy of the ball and the arm are simplified to be:

$$KE_b = \frac{7}{10}mv^2 \tag{11}$$

and

$$KE_a = \frac{1}{2}(I_{cm} + mr^2)\frac{v^2}{r^2} \tag{12}$$

The inertia of the arm was calculated using Solidworks, a modeling software.

Our final energy balance equation is as follows:

$$(PE_s + PE_a + PE_b)_i = (KE_a + KE_b + PE_s + PE_a + PE_b)_o \tag{13}$$

or

$$(\frac{1}{2}\not{V}E\epsilon^2)_s + (mgh)_a + (mgh)_b = (\frac{1}{2}I_a\frac{v^2}{r^2})_a + (\frac{7}{10}mv^2)_b + (\frac{1}{2}\not{V}E\epsilon)_s^2 + (mgh)_a + (mgh)_b \tag{14}$$

In order to determine the distance the ball travels the kinematic equation representing distance as a function of velocity is used. Therefore the equation (13) is solved for velocity.

For the transducer to read the distance the ball travels a relationship between the voltage read from the Wheatstone needs to be related to the distance the ball travels. The relationship between strain and voltage seen in equation (2) requires a strain amplification factor A.

The strain amplification factor is determined at a predetermined strain. In this case the total strain felt between the initial and final positions of the beam/spring is $2000\mu\epsilon$ and $3000\mu\epsilon$. The amplification factor is computed by solving the energy balance equation for distance where distance is equal to the product of an amplification factor A and the measured strain in the beam. This is true only when geometric properties of the system are measured both initial and in the final position.

3 Experimental Apparatus and Procedures

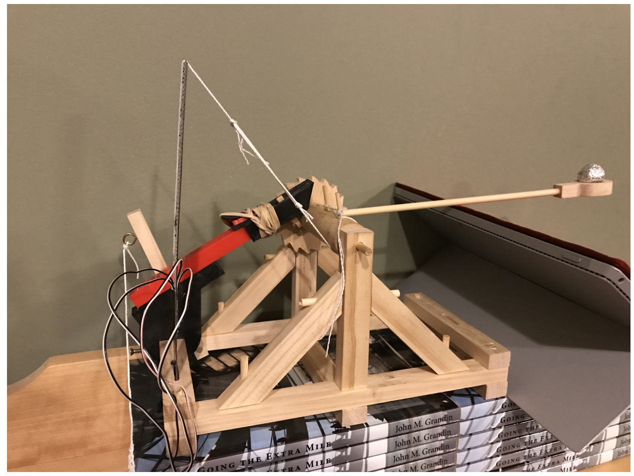

Figure 1: The Da Vinci catapult fully assembled with all modifications

A Da Vinci Catapult, based on drawings from Da Vinci's Codex Atlanticus, was used in this experiment. The apparatus consisted of 21 parts, glued together according to the instructional packet with a few modifications for this lab. For the purpose of this experiment, the bamboo tension arms were replaced with a steel cantilever beam, acting as a spring. In this case the 1095 spring steel is used due to its large elastic range and inherent spring charismatics. The equations mentioned above relating to the spring/beam hold true only within the elastic range of the material. The design calls for two tensioning beams, for this experiment only one is used. The beam is mounted in the same location as the afore mentioned bamboo beam and attached, at its other end, to the axis of rotation shown by a main drum via a string. An additional stopper was installed in order to allow a swing of the arm up to a perpendicular position relative to the ground/datum for the catapult. This is to ensure that the ball's initial travel is along the horizontal axis. When the swing arm is at its perpendicular position, the spring is at its default deflection. As the arm is primed, the change in position of the main drum will engage the spring to its fully loaded position.

Once the catapult was assembled with the correct modifications, the next step is to determine the configuration and location of the strain gauges so that strain due to bending is measured. For a rectangular cantilever beam the strain gauges will be most effective at the base of the cantilever beam as this is where the strain gauges will be most sensitive. The difference in resistance is due to the tension or compression of the zig-zag formation relative to the strain gauge terminals. Therefore, they should be placed on the beam so that deflection is felt due to deflection. When using a full Wheatstone bridge in conjunction with strain, two sensors are used to read tension and two read compression. As deformation varies along the beam it is critical that the gauges are placed at the same cross section relative to the base. This results into sets of strain gauges mounted across and next to each other. Relative to the base of the catapult the terminals could either face up or down. In order to avoid collision and stress due to unwanted wire movement the strain gauge terminals are place upwards to allow the wired to be taped along the beam.

Now that appropriate gauge locations and orientations have been determined the strain gauges may be attached to the spring/beam. To prepare the spring, a fine sandpaper was used to smooth out the surface. Next the surface is treated and sanded with Conditioner A. Once this step is complete Neutralizer N5 is applied and quickly, with one motion, wiped clean using a gauze. The surface is now ready for the strain gauge. When applying the strain gages, it is vital that no hand oils come into contact with the gage. Using tweezers the strain gauge is placed on a separate clean surface (Ex: glass plate or strain gauge container). The strain gauge is placed on this surface with the same side up as desired to be up on the device. Carefully, as not to touch the strain gauge with your hand, cover then peel up (at a low angle) the segment of tap on the gauge so that the gauge sticks to the tape. Next align the tape on the desired surface so that when the tape is fully applied the strain gauge sits in its desired position. However do not yet apply the portion of the tape holding the strain gauge. Apply a small amount of Catalyst B 200 to the back of the strain gauge and allow to dry. When dry apply a small amount of M-Bond at the bottom of the strain gauge. To apply the strain gauge firmly press the strain gauge onto the surface starting at the base and moving along the surface. This is done in order to distribute the bonding agent evenly along the bottom of the gauge as well as to avoid wrinkles in the gauge. Using your finger, apply even pressure and allow for the bonding agent to cure, the surface of the gauge may become hot to the touch. The tape and strain gauge is removed from the surface again using a small angle in attempt to not disturb the adhesive grip on the strain gauge. The residual adhesive is removed from around the strain gauge using a razor. This is repeated for the remaining 3 strain gauges to their desired location. At this point, the strain gauge was ready for connection of the lead wires.

Figure 2: 3D printed bumper

With the strain gauges mounted, the lead wires are now attached to the gauge terminals. Due to the use of 4 strain gauges, a total of 8 terminals require wires, 2 per gauge. A small gauge wire pair is selected, and 4 segments (each with 2 wires) of equal length is cut. It is important that the wires are equal in length, large differences in length will result in inaccurate resistance readings as a significantly longer wire has more resistance. At each end the two wires are separated and strip back a small amount of insulation. Using a soldering iron set to the correct temperature, solder, and flux apply a small amount of solder to each of the terminals as well as the exposed wire ends. Now that all surfaces have solder, carefully attach the wires to the terminals using the soldering iron. It is important that any excess or exposed wire is not touching any unwanted surfaces. It will also prove beneficial to consistently attach the wires according to their color for each gauge. The gauges and wires in this state may be fragile, taping the wires to the beam may provide strain relief to the terminals.

With the strain gauges mounted and secured the beam is now installed onto the catapult so that no stress is felt on the beam. The beam, containing wired strain gauges is ready to be connected to the transducer completing the full Wheatstone bridge. Twist or solder the remaining ends of the strain gauges such that a compression terminal is connected to the polar opposite end of a tension gauge. Repeat this step for the remaining gauges, the end result should have the tension and compression gauges wired opposite from one another. The connected wires are then labeled: P+, S-, P-, and S+. Those were connected to slots designated on the strain indicator. P+ is connected to S-, S- is connected to P-, P- is connected to S+, and S+ is connected to P+. Once the strain gauges are connected to the strain indicator, the system is balanced to zero by pressing the balance button on the strain indicator.

The objective of this experiment is to calculate the distance the projectile will travel when launched from the model Da Vinci Catapult. Now that construction is complete the transducer settings need to be inputed. The transducer reading a full Wheatstone bridge requires the strain gauge factor which is provided by the manufacture to be $2.100 \pm 0.5\%$. Next, the units of the transducer are set to read distance and the amplification factor found in section 2.2 is applied. The transducer is now operational.

Figure 3: Installed strain gauges with relief tape

The system is balanced when the beam has a zero deflection position. The spring is now attached via a string to the barrel of the catapult. Due to the insignificant spring force within the spring, the system must be preloaded. The spring is preloaded to the strain determined by the amplification factor when the catapult is primed. When the catapult is at rest in the launched position the strain in the beam should read 0, if it varies by more than $100\mu\epsilon$ it may be appropriate rebalanced the transducer at this location.

Five trials of this experiment will be run per two different values of strain. The microstrain is the deflection at the maximum deflection of the beam. Five trials will be run at $3000\mu\epsilon$, as well as at $4000\mu\epsilon$. Because the relationship between strain and distance the ball travels is linear, an increase in strain increases the ball distance. The transducer is then set to read linear displacement with the appropriate amplification factor

In order to calculate the amplification factor, the catapult must be launched at both $3000\mu\epsilon$ and $4000\mu\epsilon$. The amplification factor pertains to the importance of the energy balance equations for the system and the kinematics. The distance traveled is now directly proportional to the value of strain. This value isn't arbitrary. It is calculated using various potential, rotational, kinetic, and strain energies. As a result two steps are required before the experiment may be carried out. First all pertinent heights, masses, and other geometric measurements must be taken in respect to the flat ground/datum. Second, a trial must be run because the strain at maximum deflection is needed as well as the time required for the ball to travel from the free flying initial position to the ground.

The procedure for launching the catapult is as follows: balance the strain indicator to zero microstrain when the beam is initially deflected. The beam is then put to maximum deflection (i.e $3000\mu\epsilon$ or $4000\mu\epsilon$). At this point the transducer is set to read a predicted displacement. Now, the arm is retracted and the projectile loaded. The strain indicator reads the distance the ball will travel at the maximum deflection. That value will be recorded. A group member will videotaping the release of the catapult and the impact location of the projectile, so that the distance the ball travel can be evaluated as well as the time. Once the ball is launched from the catapult, the catapult will be set back to its initial conditions. Five trials in total should be recorded at both strain values.

A comparison is then made between the distances observed on the video with a reference ruler versus the distance displayed on the strain indicator. Statistical analysis, such as mean and standard deviation will help determine how accurate the strain indicator is at reading the distance the ball traveled.

4 Presentation of Results

The results shown below are given when the difference in strain between the loaded and launched positions of the catapult are $3000\mu\epsilon$, as well as at $4000\mu\epsilon$.

As stated in section 2.2 the amplification factor at each strain range is calculated using a verity of physical properties given by table 6 see appendix. Table 1 shows the resulting Amplification factors calculated using the physical properties and the energy balance equation.

Table 1: Amplification Factors at a given strain

Strain ($\mu\epsilon$):	Amplification Factor A:
$3000\mu\epsilon$	6825
$4000\mu\epsilon$	6960

Table 2 shows the results shown by the transducer as well as the accrual results measured at a bending strain of $3000\mu\epsilon$.

Table 2: Distance of travel at $3000\mu\epsilon$

Bending Strain = $3000\mu\epsilon$		
Trial:	Transducer Value (in):	Measured Value (in):
1	33.198	34.13
2	33.054	33.75
3	32.950	32.82
4	33.573	33.90
5	33.104	33.25

Table 3 shows the results shown by the transducer as well as the accrual results measured at a bending strain of $4000\mu\epsilon$.

Table 3: Distance of travel at $4000\mu\epsilon$

Bending Strain = $4000\mu\epsilon$		
Trial:	Transducer Value (in):	Measured Value (in):
1	30.00	28.86
2	30.05	31.90
3	30.07	29.25
4	30.10	30.75
5	30.03	29.75

Table 4 shows the standard deviation and mean values obtained between the transducer and measured values for $3000\mu\epsilon$ and $4000\mu\epsilon$. Initial observations may show that given the large difference between the standard deviations in both strains, this system is not yet an effective tool.

Table 4: Standard Deviation and Mean values

	Bending Strain = $3000\mu\epsilon$		Bending Strain = $4000\mu\epsilon$	
	Transducer Value:	Measured Value:	Transducer Value:	Measured Value:
Mean:	33.1758	33.57	30.05	30.102
Standard Deviation:	0.02394	0.5291	0.0381	1.2293

5 Uncertainty Analysis

Uncertainty in this experiment comes from a variety of sources. There are two major apparatuses used in this experiment: Model 3800 Strain Indicator and strain gauges used in the application of a Wheatstone Bridge. Inaccuracy in the experiment is caused by these tools. When considering these two devices, it is observed that the strain gauges have the most room for error.

The strain gauges, reading in micro-strain, placed in the configuration that measures bending strain at the base of the cantilever spring/beam. The strain gauges, applied as the resistor elements of a Wheatstone bridge, utilize electrical resistance. The transducer, a tool containing a Wheatstone bride circuit capable of measuring force, torque, pressure and more. The acceptable accuracies of the strain gauge are +/- 0.2 %. These strain gauges are general purpose load cells. The load cell has a calibration inaccuracy of 0.50%. The zero-balance, when the strain gauges are being balanced with the strain indicator, have errors within +/- 5 %.

The transverse sensitivity of a strain gauge refers to the behavior of the gage in responding to strains which are perpendicular to the primary sensing axis of the gauge. For the strain gauges used in this experiment, the error in transverse sensitivity is +/- 0.2 %. The error is generally small, especially due to the known Poisson ratio of the 1095 steel, and its uniaxial stress state. A gauge factor is necessary for these strain gauges as there were no wires pre-attached. A gauge factor is the load cell's strain sensitivity. The gauge factor is given by the manufacturer as 2.1 +/-0.5%.

For the apparatus, the strain gauges were placed at the bottom of the beam. The beam triggers the catapult to launch, thus experiences the deflection. The strain gauges are strategically placed at the bottom of the beam because this location experiences the least deformation. The less deformation the strain gauges experience, allows for an increased sensitivity in the readings of the distance the ball travels as predicted by the strain indicator.

The strain indicator read values in a fourth order of magnitude. In this experiment, when the beam was fully deflected there was fluctuation by about $\pm$ 30 micro-strain when the beam was in a stagnant position. This is most likely due to noise within the system, as well as noise within the wires. This fluctuation falls within the 5% zero-balance range.

Table 5 shows a comparison between the transducer and measured results at $4000\mu\epsilon$ and $3000\mu\epsilon$. These results are shown for a desired confidence interval of 98%. A 2% deviation is an accepted amount of variance.

Data focused on in this experiment shows the compares data obtained from the strain indicator and the data from the video recording of the experiment. All data is representative of the distance the ball traveled in inches. When the beam was deflected to $3000\mu\epsilon$, five trials were performed, launching the ball from the catapult. The amplification factor was applied to the strain indicator and all output values are in inches that the ball will travel. For the $3000\mu\epsilon$ trial, the mean of the difference between the strain indicator values and the actual values of distance traveled is 0.4467 with a standard deviation of 0.35114. Based on these results, it is evident that the strain indicator was not very accurate in it's readings as the standard deviation is only slightly smaller than the mean. A 98 % confidence interval states that one is 98% confident that the difference between the strain indicator value and the

Table 5: 98% Confidence Interval Values for given strain

	$3000\mu\epsilon$	$4000\mu\epsilon$
Mean	0.4467	0.945
STD	0.35114	0.59083
alpha level	0.01	0.01
Degrees of Freedom	4	4
t value	3.747	3.747
Standard of error	0.15703	0.2642
SE*t value	0.58829	0.9906
Lower end of range	-0.1417	-0.04506
Upper end of range	1.03509	1.93506

actual value will fall between -0.1417 in and 1.03509 in. Note that negative values are not possible for the parameters of this experiment. When the accurate measurement is obtained, there is a significant range for which the strain indicator value will lay, or vice versa. In this case, often the strain indicator underestimated the actual distance of the ball.

For the $4000\mu\epsilon$ trial, five trials were performed, and the data analyzed is the difference between the values produced from the strain indicator and the actual distance that the ball traveled. The mean of the difference of the two values is 0.945 in with a standard deviation of 0.59083 in. Again, these results are not representative of an accurate strain indicator reading due to the standard deviation being more the half of the size of the mean. This suggests that the values are spread out. Note th at an outlier analysis was performed on both data sets ($3000\mu\epsilon$ and $4000\mu\epsilon$), and no outliers where found. The confidence interval states: one is 98% confident that the difference between the strain indicator readings and the actual readings of the distance traveled is -0.04506 in and 1.93506 in. Note that negative values are not plausible in this experiment. The values read on the strain indicator have a significant range in which the actual distance that the ball traveled will fall. In this case, the strain indicator often overestimated the amount that the ball will travel.

Note that when the maximum deflection of the beam is at an increased strain, there is less variance in the strain indicator values being produced by the machine. This means that as strain increases, the sensitivity in the system increases. Increased sensitivity at higher strain means that the repeatability of this experiment is better at higher strain values, thus increased deflection. To minimize some of the inaccuracies mentioned in this experiment, performing this experiment at increased strain is advised. Due to the zero-balance error, the fluctuation even at its greatest is negligible because it was within 5% of the values being produced by the transducer. However, less fluctuation is desired in terms of obtaining a value that is close to the measured value.

6 Discussion of Results

The purpose of this experiment is to evaluate the accuracy of the combination of strain gauges and a strain indicator. Strain gauges in the form of a Wheatstone Bridge have a multitude of capabilities in terms of measurement and have a wide variety of applications. For this experiment, the combination of a Wheatstone Bridge and a strain indicator determined how far a ball will travel when launched by a Da Vinci catapult. The strain indicator produced the distance value based on an amplification factor, determined by a relationship between distance and strain. The strain indicator value was compared to the actual distance the ball traveled. This distance was obtained by taking a video of the catapult being launched with a measuring tape in view as a reference.

In conclusion, for the purpose of this experiment, the strain indicator did not accurately determine the distance that the ball would travel. Five trials were conducted at two different maximum deflection values: $3000\mu\epsilon$ and $4000\mu\epsilon$. For the $3000\mu\epsilon$ trials, the first step was to calculate an amplification factor. The process of calculating the amplification factor can be found in section 2.2. The amplification factor for these trials is 6825. This value is inputted into the strain indicator and the units are changed so that the machine is now reading values in inches. Analysis of this data was performed in terms of the difference between the transducer value and the measured value. As well, the mean and standard deviation were taken for the transducer values and the measured values separately. The mean value that the transducer determined at 3000 micro-strain for the distance the ball travels is 33.178 in with standard deviation of 0.2394. The mean measured value is 33.5700 in with a standard deviation of 0.5291 in.

The analysis that determined the accuracy of the system is the confidence intervals calculated based on the difference between the transducer value and the measured value. A confidence interval determines a range for which a specified will fall between confidently. The difference between the transducer values and the measured values were taken, and an outlier analysis was performed resulting in no identified outliers. The mean of the difference of the values is 0.4467 in with a standard deviation of 0.35114 in. Immediately, these results are not representative of an accurate strain indicator reading because the standard deviation, the average distance from the mean, is only slightly smaller than the average value. This suggests that there is large variance between the transducer values and the measured values, which is not indicative of an accurate system. The confidence interval states that one is 98 % confident that the difference between the transducer values and the measured values is between -0.1417 in and 1.03509 in. Note that negative values are not possible in this experiment, as the ball can not be launched in the negative direction. Again, this is a wide span of values that say the measured values will be in that range in relation to the strain indicator value. The smaller the the difference value, the more accurate the system is.

For the 4000 micro-strain trials, the same analysis was performed to determine the accuracy of the strain indicator system at an increased strain. The average distance traveled by the ball, according to the transducer values is 30.05 inches with a standard deviation of 0.0381 inches. Note that this standard deviation is small, the fluctuation in at 4000 micro-strain for the transducer was not huge, representing consistent results. The actual average distance that the ball traveled is 30.102 inches with a standard deviation of 1.2293 inches.

This standard deviation is about 10% of the mean. There is variance in the actual distance the ball traveled, despite the strain indicator having little fluctuation.

A confidence interval was calculated for the 4000$\mu\epsilon$ values. First outlier analysis was performed on the difference between the transducer values and the actual measurement values, and there were no conclusive outliers. The difference between the transducer distance values and the measured distance values have an average value of 0.945 inches with a standard deviation of 0.59083 inches. Looking at these values, it can be said that the strain indicator does not accurately read the actual distance that the ball travels. The standard deviation, the data points average distance from the mean, is over half the size of average of the values. This means that there is significant variance between the transducer values and the actual measurement values, thus the strain indicator is not accurate in measuring the distance that the ball travels. The confidence interval states that one is 98% confident that the difference between the transducer and measurement values for which the ball travels, is between -0.04506 and 1.93506 inches. Note that the negative values in this experiment are not plausible as the ball was only launched in the positive direction. This is a wide range of values for which there can be a difference between the transducer value and the measured value. The strain indicator can vary from the actual reading for the distance the ball traveled by that range, and vice versa. That range is large and due to the mean and standard deviation of the system, the strain indicator was not accurate in reading the distance that the ball traveled.

In terms of the strain indicator itself, a conclusion can be made in terms of the amount the beam was deflected and the variance of the system in general. When the beam was in more tension, the strain at maximum deflection was higher, there was less variance in the values that strain indicator determined. When the strain indicator was at 3000$\mu\epsilon$, the values that the strain indicator produced fluctuated by about 30$\mu\epsilon$. This is most likely due to noise in the strain indicator and/or the wires connecting the strain gauges to the strain indicator. However, when the beam is in more tension and thus the strain value is higher, the strain indicator values didn't fluctuate much at all. This means that as the tension in the system increases, the sensitivity increases as well. Repeatability of this experiment is more consistent at higher strain readings. Although the fluctuation is within the zero balance error, it would still be advised to perform the experiment at increased maximum strain values. The fluctuation makes it difficult to determine the actual reading from the strain indicator.

7 Conclusions and Recommendations

In conclusion, this experiment that the strain indicator could no accurately determine the distance that the ball traveled. In this experiment, two different deflections were tested and five trials were run per deflection value. From the collection of data, conclusions can be made on the accuracy of the system, the impact of deflection on the experiment, and changes to be made so that the experiment can be repeated with more successful results.

Confidence intervals were performed on the difference between the measured distances and the transducer predicted distances that the ball would travel. The difference between the values was the main source for analysis as the goal is to establish the difference between the values as close to zero as possible. The standard deviation of all the data sets also reflects on the variance of the experimental values collected.

For the experiment performed at 3000 bending $\mu\epsilon$, the transducer values had an average distance traveled of 33.1758 inches with a standard deviation of 0.02394 inches. This standard deviation is small, about .072 % of the mean value, which signifies little variance in the data values collected. The average distance traveled for the measured values is 33.57 inches with a standard deviation of 0.5291 inches. This standard deviation is also fairly small, only being 1.57% of the mean value. That percentage statistically insignificant, which means that there is also little variance in these data values collected. The 98% confidence interval for the difference between the transducer and measured value states that one is 98% confident that the average distance between the values will fall between -0.1417 inches and 1.03509 inches. This range is not comprehensive an accurate system. This can be said because the average value of the difference between the two distance values is 0.4467 inches with a standard deviation of 0.35114 inches. The standard deviation is 78.6 % of the average value. This signifies much variance in the difference of the values. This then makes the system inaccurate in reading the distance because the transducer never consistently matched the measure values, or was close.

The experiment performed at 4000 bending $\mu\epsilon$ had an average measure distance traveled by the ball predicted by the transducer of 30.05 inches with a standard deviation of 0.0381 inches. This standard deviation is small, which signifies little variance in the data values obtained from the transducer. The average distance traveled obtained from the measured values is 30.102 inches with a standard deviation of 1.2293 inches. This standard deviation is the largest of all data sets collected. There is variance in this data collected due to the standard deviation being somewhat high. The 98% confidence interval for the difference between the transducer and measured value states that one is 98% confident that the average distance between the values will fall between -0.04506 inches and 1.93506 inches. The mean for the difference between the vales is 0.945 inches with a standard deviation of 0.59083 inches. This, like the previous trial, does not signify the transducer being comparable to the measured distance. A standard deviation that high, as well as a range, that large says that the difference between the values wasn't consistent or consistently small.

It may be observed that the there was a larger variance in the measured values opposed to the transducer value. This suggests that there were unknown/unconsidered variables affecting the travel of the ball. It is not possible to consider all variables. A more accurate model might serve to close the gap between accrual and measured results. An analysis of dynamic variables may be used to apply a range of accuracy.

If this experiment were to be repeated some alterations to the procedure and process are recommended. The ball used in this experiment was made of aluminum foil. It would be advised to use a marble or a small metal ball as it would have more mass consistent, air friction would be less of a factor, and the ball wouldn't be deformable. As well, the catapult system was a kit with some general flaws in it's design. The friction between the pins may have caused disturbance in the system. Which is why the strain indicator over or under predicted the distance that the ball will travel. The last alteration to the procedure of this experiment would be to recalibrate the strain gauge after each trial. This may reduce the range of values measured by the transducer. These factors that were not changed in the original experiment may have impacted the accuracy of the strain indicator system in terms of reading the distance that the ball travels.

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Bibliography

[1] Carl Ernst Rousseau. *Manual: MCE 313 - Project, Transducer Design.*

[2] Wikipedi: *Catapult,* March 28, 2018.
`https://en.wikipedia.org/wiki/Mean`

[3] Wikipedi: *Leaf spring,* Febuary 23, 2018.
`https://en.wikipedia.org/wiki/Leaf_spring`

[4] Transducer Techniques: *Quarter Bridge, Half Bridge and Full Wheatstone Bridge Strain Gauge Load Cell configurations.,* `https://www.transducertechniques.com/wheatstone-bridge.aspx`

[5] Omega Engineering: *Strain Gauges, An introduction to Strain Gauges,* `https://www.omega.com/prodinfo/straingages.html`

[6] Carl Ernst Rousseau. *Manual: MCE 313 Lab 1, The Instron Testing Machine Load Deflection Behavior of Vibration Isolators.*

[7] WikiPedia. *Hooke's Law*
`https://en.wikipedia.org/wiki/Hooke27s_law` Jan 2018.

[8] Arun Shukla and James W. Dally. *Instrumentation and Sensors for Engineering Measurements and Process Control*

[9] William Palm III. *System Dynamics Third Edition*

[10] Intertechnology Inc. *3800 Wide Range Strain Indicator* 3800 Wide Range Strain Indicator 1984.

[11] Correlated Solutions *Reference Manual* Vic-3D 2010 2010.

9 Appendices

Table 6: Physical Properties

Spring/Beam					
Mass	36.37	g			
Total Length	9.61	in	0.244	m	
Width	1.246	in	0.032	m	
Thickness	0.0251	in	0.00064	m	
E	207	GPa			
Catapult arm Components					
Pine	Density	420	kg/m^3		
Barrel	Mass	27.33	g		
	Length	3.756	in	9.5402	m
	Diameter	0.965	in	0.0245	m
Gear	Mass	14.53	g		
	Thickness	0.318	in	0.0081	m
	Diameter Major	2.769	in	0.0703	m
	Diameter Minor	2.311	in	0.0587	m
Cup	Mass	6.14	g		
	Hole Depth	0.405	in	0.0103	m
	SG from Base	0.305	in	0.0077	m
Axial Pin (x2)	Mass	0.51	g		
	Length	1.569	in	0.0399	m
	Diameter	0.295	in	0.0075	m
Set Pin	Mass				
	Length	0.848	in	0.0215	m
	Diameter	0.295	in	0.0075	m
Bamboo	Density	400	kg/m^3		
Arm	Mass	5.59	g		
	Length	9.422	in	0.2393	m
	Length exposed	8.85	in	0.2248	m
	Diameter	0.235	in	0.0060	m
Arm System					
Mass	54.46	g	0.1200636052	lb	

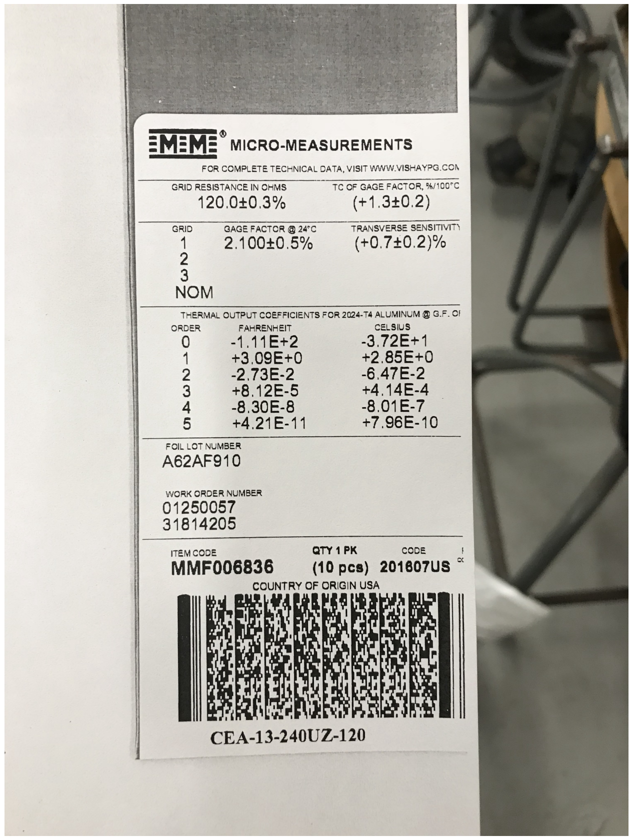

Figure 4: Strain gauge information provided by Manufacture

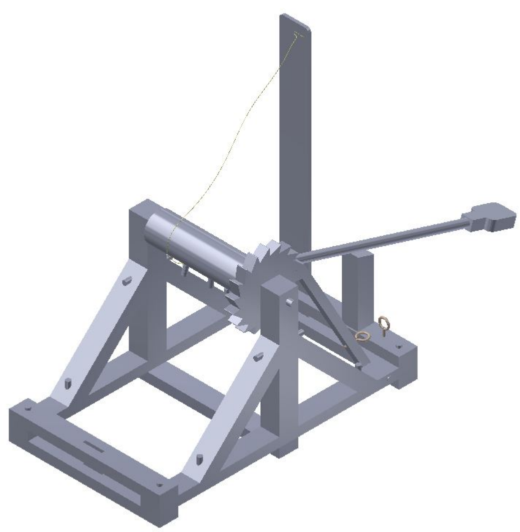

Figure 5: Drawing of the Da Vinci catapult used