Statistical Analysis of the Instron Testing Machine Load Deflection Behavior of Vibration Isolators
MCE 313 - Lab 3

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Abstract

Statistical analysisis is a key part of the experimental procedure because the application interprets and communicates research findings to a higher degree. The statistical analysis applied to these data sets included: comparative analysis, 98% confidence interval, outlier analysis, mean, and standard deviation. Two sets of data were used in this report: the experimental data from the Instron Testing Machine Load Deflection Behavior of Vibration Isolators lab experiment, and a set of given data. The Instron 5582 was used to gradually compress a Coiled spring isolator at increments of 0.05 inches until the total displacement reached 0.6 inches. At each increment, the force applied was measured in pounds force(lb_f). The stress-strain curve was successful in representing the Coiled spring isolator, and confirmed it to be linear, with a maximum load of $750 lb_f$ resulting in a deflection of 1.06488 inches. A 98% confidence interval was obtained for both data sets. For the experimental data set, one is 98% confident that the mean of that data set is within the range of 697.7362 lb_f/in to 710.7598 lb_f/in. The given data set had a 98% confidence interval of values that ranged from 690.0774 $lb_f/in$to 700.6976 lb_f/in. One is a 98% confident that the average spring constant fall between those stated ranges. A comparative analysis of the two data sets was performed. The null hypothesis states that no statistical significance exists between the two sets of data. Once comparative analysis was performed, the null hypothesis was rejected. Indicating that there is statistical significance between the two data sets, and variation exists.

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Nomenclature

Symbols & Units

δ	Absolute value of the difference between any point and mean value
δ_{max}	Max deviation
$\overline{x}$	Mean value
τ	Thompson's value
b_i	Initial load
dx	Change in deflection
F	Force developed by the spring
H_1	Alternative hypothesis
H_o	Null hypothesis
k	Spring stiffness
k_f	Mean stiffness of the frame
n	Number of observations in the data set
P_i	Particular load
S	Standard deviation
t	t-distribution value for a data set
t_0	t-distribution value for the comparative analysis
v	Degrees of Freedom
x	Deflection
x_i	Any single observation
y_f	Frame deflection at a load

1 Introduction

The purpose of performing statistical analysis on data sets is that it allows those performing the experiment and analyzing the data to make experimentally backed conclusions and understand uncertainty in those conclusions. Statistical analysis has several uses including: understanding the nature of the data, the relation of the data to the underlying population, and to identify trends. In this analysis, the following analyses were performed: outlier analysis, standard deviation, mean, 98% confidence interval, and comparative analysis. There are two data sets being analyzed. One data set is the experimental data provided from the Instron Testing Machine during the Load Deflection Behavior of Vibration Isolators lab performed prior to this analysis. The data from this lab shows only the spring constant values, k, of the coiled spring isolator, which were discovered using corrected deflection values. The second set of spring constant data is given, and is representative of 10 experiments performed using the same apparatus and procedure [2].

The general procedure of this experiment is as follows: the Instron gradually compresses a Coiled spring isolator at increments of 0.05 inches until the total displacement reaches 0.6 inches. At each increment, the force applied was measured in pounds force(lb_f) [1]. The stress-strain curve was successful in representing the Coiled spring isolator, which was confirmed to be linear, having a maximum load of $750 lb_f$ resulting in a deflection of 1.0648 inches. Those values are concurrent with the Data obtained by the manufacture.

On both the experimental and given data sets an outlier analysis is performed. This process will eliminate values that have a low probability of occurrence [8]. Once the outlier analysis is performed for both data sets, the mean and standard deviation will be recalculated individually. A confidence interval is obtained for both data sets. The results of those are as follows: one is 98% confident that the spring constant value will fall within the range 697.7326 lb_f/in and 710.7598 lb_f/in, for the experimental data set. One is 98% confident that the spring constant value will fall within the range of 690.0774 lb_f/in to 700.6976 lb_f/in.

Lastly, a comparative analysis was performed on both the experimental and given data sets. The comparative analysis states whether or not the data sets experience any variance between one another and whether that difference is statistically significant. In hand, it determines whether the difference between the two means is significant, or due to variance. The null hypothesis used said that there is no statistical significance between the two data sets [8]. The analysis performed rejected the null hypothesis, stating that there is statistical significance between the two data sets. Variance occurs within the experimental and given data. As a result of the variance, one is able to say that the spring constant for the experimental data is higher than the spring constant value from the given data set.

2 Theory

The purpose of lab 1 was to verify the stiffness of a given spring in accordance with Hooke's Law [1,7]. Hooke's Law states that the force needed to extend or compress a spring is dictated by the stiffness or spring coefficient and some linear distance. Hooke's Law can be written using the equations below:

$$F = kx \tag{1}$$

or

$$F(x) = kdx + b_i, \tag{2}$$

where F or $F(x)$ is the force or load developed by the spring, k represents the spring coefficient or stiffness and x or dx represents deflection and change in deflection, respectively. The b_i value represents any initial load [7].

The experimental results for the linear spring showed that the spring stiffness, k, obtained was close to the spring stiffness provided by the manufacture. In an arena where more precise values are required with higher confidence you must complete a variety of statistical analyses. Statistical analyses are used in research to interpret and communicate research findings. These findings may or may not support the hypotheses. Statistical analysis can also support the methodology and conclusions.

The mean of a data set refers to the central tendency of a set of data [6]. Finding the mean is the average or central value obtained. This is computed by dividing the sum of the data by the number of data points as seen here:

$$\overline{x} = \frac{1}{n}(\sum_{i=1}^{n} x_i) \tag{3}$$

where the mean value represented by $\overline{x}$. The value n represents the number of observations in the data set and x_i is a singular observation or data point. This central value increases its confidence when n is large.

Deviation of a data set indicates difference between a data point and the set's mean. Deviation is defined as:

$$\delta = |x_i - \overline{x}| \tag{4}$$

where the deviation, δ, is the absolute value of the difference between any point and the mean value. The total amount of deviation from a set's mean is known as Variance. The variance may be obtained by following equation:

$$S^2 = \frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{n-1} \tag{5}$$

Variance is represented by S^2. The sum of the mean value, represented by $\overline{x}$, and the sample points, x_i, are squared. The subsequent value is that sum divided by $n - 1$ or v, representing the degrees of freedom in the set. The square root of your variance gives the standard deviation. Standard deviation represents the average variance of a data set; this can be understood as the mean of the variance. Therefore, a low standard deviation indicates low variance and more precision (not accuracy) in a data set where a higher value suggests larger deviation. In order to find the standard deviation, use:

$$S = \sqrt{S^2} \tag{6}$$

Standard deviation is used indicate the mean variance for a data set is spread out from the mean. The standard deviation may be used to identify an expected value range.

In any data set, there is a risk of data points that lay outside of accepted boundaries. In order to state findings accurately, it is important to neglect these outlying data points. This is done through an outlier analysis. A data point is an outlier when it's deviation lies outside of a certain threshold defined by the following equation.

$$\delta_{max} > \tau S \tag{7}$$

The value δ_{max} is the maximum deviation. This must be larger than product of the standard deviation found in equation (6) and τ value. The value τ or Thompson's value is obtained using a table found in the appendix [9].

The outlier analysis only ever identifies one value. This value is then neglected. After each iteration a new max deviation and τ must be identified with the revised data set. Subsequently, the mean and standard deviation must also be recalculated for the revised set.

The results from lab 1 were used to find the spring stiffness coefficient, k. The observation was made that the experimental results were similar to the stiffness provided by the manufacturer. When a manufacturer makes a claim, it says that every time our device is used, it performs a certain way. Unfortunately, this is only a hypothesis. Often, there is a risk of variance. If the claim were true, all experimental trials would be identical. In order to make a claim which includes this risk factor, the statement must include a confidence interval. A confidence interval allows the hypothesis to include deviation from the mean value (in this case expected value) with a designated % level of confidence represented by a range. The % amount of confidence level is represented by α. A range including an above or below value is known as a 2-Sided. This hypothesis follows the equation below:

$$\mu = \overline{x} \pm t_{\alpha/2}\frac{S}{\sqrt{n}} \tag{8}$$

The value μ gives the maximum and minimum values for your confidence range. The mean, $\overline{x}$ is accompanied by the value of accepted deviation. The deviation is the product of $t_{\alpha/2}$, found on the t-distribution table, and the quotient of the standard deviation and the root of the data points. The t-distribution is used due to small sample size. A graph of the t-distribution values about the mean gives a bell curve. A Gaussian or normal distribution my be used to project results for larger sample size. The maximum of this curve is at the mean value. Area under the curve represents the sum confidence. The value α represents the sum area under that curve between the standard deviation and the respective μ value. The value $t_{\alpha/2}$ is found by relating $\alpha/2$ and your degrees of freedom on a t-distribution table.

Given a good understanding of a set or sets of data, it may be appropriate to compare a data set with a second set of data. This may provide a variety of information and/or answer specific questions. For instance; if experimental method varied, which method was more accurate; whether or not if two different formulations of the same product give equivalent results in order to test repeatability; and/or to observe a trend or identify claims relating to the sample error. This information may be gained through a statistical branch called comparative analysis. In this lab, a statistical technique referred to as hypothesis testing is employed. The values obtained above represent a single set of data. A comparative analysis compares many of the values obtained above in order to better understand and identify any relationships or lack there of.

To begin hypothesis testing, two mutually exclusive hypotheses need to be made based on the purpose of the comparative analysis. The null hypothesis, H_0, states that your hypothesis is true. The alternative hypothesis, H_1, states that your hypothesis is false thereby rejecting the null hypothesis. In this experiment the hope is to understand whether there is deviation between the experimental data and the data collected by a different group. This is evaluated by comparing the t value obtained using the t-distribution tables to a t_0 value obtained using the following equations. Note: the following equations two are for 2 sets of data.

$$t_0 = \frac{\overline{x_1} - \overline{x_2}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \tag{9}$$

Where S_p can be found using the equation:

$$S_p = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}} \tag{10}$$

If the two t values, t and t_0, are found to be equal this indicates that the null hypothesis is supported. If the two t values are not equal this indicates that variance between the two data sets is present. This can be understood as the mean value for each set exists outside of the confidence interval of the other. This may suggest that a larger interval is required.

3 Experimental Apparatus and Procedures

An Instron tester is used as our primary apparatus for testing. The Instron model 5582 (fig.1) tester, used during this experiment, is a screw driven testing device used for both compression and tension [3]. In this experiment the Instron was used to compress isolators or springs (fig.2) [1]. Compression of the isolator is done using a load cell or force measurement transducer. The load cell is attached to the cross head. Factors that affect the accuracy of the load cell are: change in temperature and change in balance as related to the applied load. In this case a load cell of 100kN was used [1] [3].

Figure 1: Instron model 5582 equipped with a 100kN load cell

The first step is to turn on the Instron, seen in Figure 1, and wait for the electronic display on the base of the machine to finish the start up sequence. Then activate the control software, Merlin, on the computer. Ensure that the proper load cell is loaded and that Merlin is reading, from left to right, "Load" measured in pounds forcelb_f and "Extension" measured in inches (in). Place the first isolator on the center of the load bearing plate so that the isolator is as centered as possible. Lower the load cell using the jog button. Release jog shortly before the load cell touches the isolator. Next, place a piece of paper on top of the isolator. Complete a paper test by wiggling the paper lowering the load cell using the wheel control until the paper is just barely able to slide out. This ensures that the gap between the isolator and cross head is minimal. Be sure to zero both the load and deflection values in Merlin before continuing. The experiment is now set up [1] [3].

(a) (b)

Figure 2: a) Coiled Spring (isolator 1) b) Linear Arch Spring (isolator 2)

For this experiment load will be applied to the isolator by lowering the cross head, there by increasing the deflection, in 0.05 inch increments. The head will read its final position after 0.60 inches of deflection. A table is used to collect load at each increment. In order to run the experiment begin to apply the load to the isolator using the scroll wheel. Slowly move the cross head down until deflection reaches 0.05 inches and record the load and deflection values. Continue lowering the cross head until the deflection reaches 0.60 inches, recording values every 0.05 inches. Once completed, press the jog up button to remove the load. Repeat this test from procedure five times per isolator in order to verify results and ensure repeatability [1] [3].

4 Presentation of Results

Table 1 is a collection of the values used to perform the statistical analysis of the compression of a coiled spring Isolator using a Instron 5582. There are two sets of numbers for the given and the experimental data. The original data set is followed by the data set with the outliers removed. An outlier analysis was performed on the original data sets to extract values that have a low probability of occurrence.

Table 1: Experimental and Given Spring Constant Values with Outlier Analysis

Given Data	Given Data Without Outliers	Experimental Data	Experimental Data Without Outliers
703.4	703.4	704.3	704.3
690.2	690.2	702.19	702.19
680.3	695.5	710.95	710.95
708.8	689.3	702.29	702.29
695.5	694.3	701.51	701.51
689.3	692.7	-	-
694.3	696.7	-	-
692.7	701.6	-	-
696.1	-	-	-
701.6	-	-	-

To determine the confidence interval, often the theory of Gaussian normal distribution is applied. The Gaussian distribution is used to estimate the mean and gives an interval estimate of where the mean will occur. The distribution is a bell shaped curve with the y-axis representing how often a measurement is likely to occur, and the x-axis representing the measurement. In the case of this experiment, the measurement is the spring constant (lb_f/in). A t-distribution was used due to the small sample size. On the curves in the figures shown below, the peak of the curve represents the mean of the data set. The shaded area on the tail ends of the curve represents the alpha level. The alpha level represents the probability of rejecting the null hypothesis. Because both intervals are calculated at 98%, the tail ends on both sides have an area of 0.01. All values necessary to calculate the confidence interval and the final interval are presented in Table 2.

Table 2: 98% Confidence Interval Values

	Given Data	Experimental Data
Mean	695.3875	704.248
STD	5.0098	3.8886
alpha level	0.01	0.01
Degrees of Freedom	7	4
t value	2.998	3.747
Standard of error	1.77123	1.73787
SE*t value	5.3101	6.5118
Lower end of range	690.0774	697.7362
Upper end of range	700.6976	710.7598

Figure 3 represents the 98% confidence interval for the experimental data after outlier analysis. The significance of this graph is: one is 98% confident that an experimental value of the spring constant will fall within the range 697.7362 lb_f/in and 710.7598 lb_f/in.

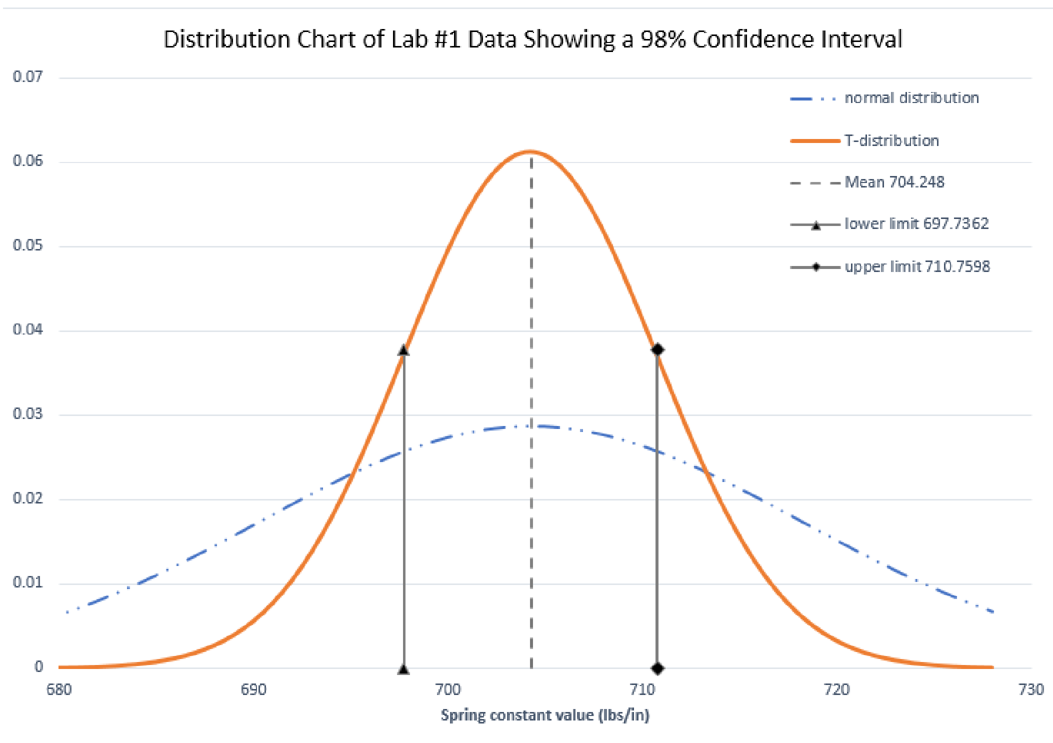

Figure 3: Distribution Table of Lab 1 Results with 98% Confidence Intervals

Figure 4 represents the 98% confidence interval for the given data after outlier analysis. The significance of this graph is: one in 98% confident that a given spring constant value will fall within the range 690.0774 lb_f/in to 700.6976 lb_f/in.

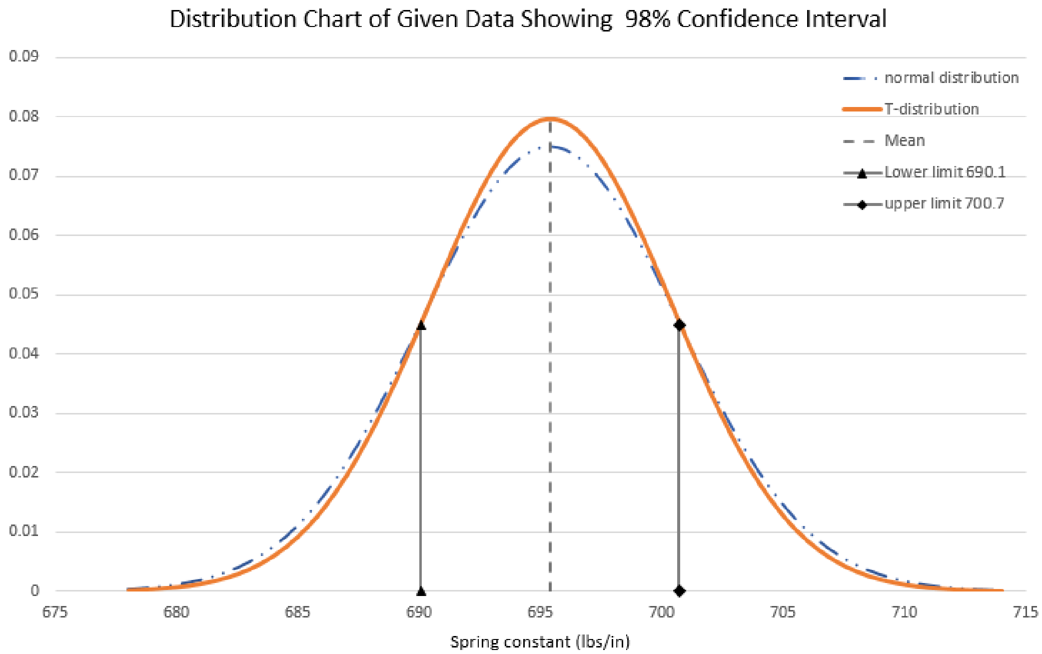

Figure 4: Distribution Table of the Given Results with 98% Confidence Intervals

Table 3 contains the variables necessary to perform a comparative analysis of the experimental and given data sets. The ultimate value that is being calculated is t_0. The calculated t value compared to a value in the t-distribution table. The value from the t-distribution table is 2.718. The result of comparing these two values, dictates whether or not there is variance between the two data sets. Since the value from the table is not equal to the calculated value of 3.354, the conclusion is that the data sets have variance between each other at a 98% confidence interval.

Table 3: Comparative Analysis Calculated Values

Experimental Data		Given Data	
$\overline{x_1}$	704.248	$\overline{x_2}$	695.3875
S_1	3.8886	S_2	5.0098
S_1^2	15.1212	S_2^2	25.09809
n_1	5	n_2	8
v	11	v	11
Comparative Analysis Results			
S_p	4.633587	t	2.718
t_0	3.354	(using table)	

5 Uncertainty Analysis

Outlier analysis is a method used to eliminate values that have a low probability of occurring when they deviate. The first step in the outlier analysis is to take the mean and standard deviation of the original data sets. Performing outlier analysis is also known as the Thompson's τ Test. A Thompson's τ table provides a value, that when multiplied by the standard deviation, equation (7), creates a threshold for how much a value can deviate.

Table 4: Outlier Analysis of Given and Experimental Data

Given Data				Experimental Data			
Original Mean	695.22	τ	1.789	-No Outliers			
Original STD	8.048	τS	14.47	-No Outliers			
1st corrected Mean	696.8778	τ	1.777				
1st corrected mean STD	6.4768	τS	11.51				
Final Mean	695.3875	τ	1.749	Final Mean	704.248	τ	1.572
Final STD	5.0098	τS	8.762	Final STD	3.8886	τS	6.1128

As seen in table 4 the Thompson's τ Test must be performed several times until there are no more outliers in the data set. Data values are not considered outliers when the deviation of a data value is lower than the τS value, the maximum deviation a point can have to not be considered an outlier. For the given data set, the Thompson's τ Test had to iterated three times in order to remove all outliers. The final mean is 695.3875 lb_f/in and the final standard deviation is 5.0098 lb_f/in. For the experimental data, after performing the Thompson's τ Test one time, there were no outliers found in the data set. So the original data set is also the final data set, with a mean of 704.248 lb_f/in and a standard deviation 3.8886 lb_f/in.

Table 5: Outlier Analysis: Given Data Values

Original Values	Deviation	New Values	Deviation	Final Values	Deviation
703.4	8.18	703.4	6.5222	703.4	8.0125
690.2	5.02	690.2	6.6778	690.2	5.1875
680.3	14.92	708.8	11.9222	695.5	0.1125
708.8	13.58	695.5	1.3778	689.3	6.0875
695.5	0.28	689.3	7.5778	694.3	1.0875
689.3	5.92	694.3	2.5778	692.7	2.6875
694.3	0.92	692.7	4.1778	696.1	0.7125
692.7	2.52	696.1	0.7778	701.6	6.2125
696.1	0.88	701.6	4.7222		
701.6	6.38				

Table 5 shows the iteration of the Thompson's τ Test in terms of the data values. The first column represents the data values and the column that follows is the deviation of the data value, which is determined by equation 4. A value that is colored red means that it was determined an outlier based on the τS value. For example, looking at the original data set of the given data, a τS value of 14.47 was determined. The spring constant value of 680.3 lb_f/in had a deviation of 14.92. Because this value is higher than the TauS value, 680.3 lb_f/in is deemed an outlier.

Table 6: Outlier Analysis: Experimental Values

Final Value	Deviation
704.3	0.052
702.19	2.058
710.95	6.702
702.29	1.958
701.51	2.738

Table 6 shows the outlier analysis for the experimental data values. No iterations of the Thompson's τ Test needed to be performed because all values in the original data set had deviations lower than the τS value. The τS value for the experimental data was 6.1128 and all deviations calculated using equation (4), were lower than this number.

For uncertainty in terms of the experimental process, the frame of the Instron 5582 is subject to compliance; a deflection under a load. Therefore, using the extension readout of the Instron 5582 to measure deflection introduces error. Specifications for the Instron 5582 found in the reference manual for users indicate that the mean stiffness of the frame $k_F = (1.46 * 10^6) lb/in$ [3, 7]. Assuming that the Instron 5582 frame deflects linearly with respect to load, a formula may be derived to determine the deflection of the frame under certain loads:

$$P_i = k_F y_F, \tag{11}$$

where P_i is a particular load and y_F is the frame defection at that load. The maximum frame deflection from recorded loads for each isolator are negligible is as shown in table 7:

Table 7: Maximum Frame Deflection Due to Compliance

Spring Coil Isolator	Linear Arch Isolator
0.00022767 in 0.0455% error	0.0002906 in 0.0581% error

A second major contributor to the uncertainty in this lab is our procedure dictating a static starting point [1]. Using a constant dynamic load would require equation (2) where dx is a function of time(t). An experiment conducted using a steady slow load rate would allow for more accurate readings. Equation (1) is used when preforming static loading however start position and moment of engagement are impossible to test for. Due to this a curve fitting process was carried out.

6 Discussion of Results

To begin the statistical analysis, an outlier analysis using Thompson's τ Test was performed on both the given and experimental data. The purpose of this test is to remove values from the data set with a low probability of occurring, specifically by observing the variance of each data value. For the given values of the spring constant, two iterations of the Thompson's τ Test had to be performed in order to eliminate all outliers. The final values of the given data used to perform the rest of the statistical analysis can be seen in Table 5. All acceptable values had a deviation smaller than the τS value calculated in table 4. For the experimental data, after one Thompson's τ Test, none of the data values had deviations larger than the τS value. Thus the original data is the final data used to perform further statistical analyses.

After performing the outlier analysis for both the given and experimental data set, the mean and standard deviation were calculated separately for both sets. For the given data set, the mean is 695.3875 lb_f/in with a standard deviation of $\pm$5.0098 lb_f/in. For the experimental data set, the mean is 704.248 lb_f/in with a standard deviation of $\pm$3.8886 lb_f/in. Using the mean and standard deviation for both data sets, along with the sample size, a 98% confidence interval was formulated. This interval is representative of the normal distribution of the data sets. This curve represents the measurement, which is the spring constant in lb_f/in, versus how often the value is likely to occur. The peak of the normal curve is representative of the true mean. For the given data the peak is at 695.3875 lb_f/in and for the experimental data the peak of the curve is at 704.248 lb_f/in.

The peak of the normal curve, or the mean of the data set is important in evaluating the confidence interval, as it is the reference point. The purpose of a confidence interval is to calculate a range of data that the mean is likely to fall within. In order to do that, one must begin by calculating the alpha level. The alpha level represents the shaped area underneath the curve. For a 98% confidence interval, the alpha level is 0.01. the alpha level is the probability of rejecting the null hypothesis. This value is pertinent in determining the t value. The t distribution table is used because the sample size is less than 30. The t value is determined by looking at the table with knowledge of the sample size and the alpha level.

By multiplying the standard error by the t distribution value, the range for the confidence interval is created. This value is added and subtracted from the mean of the data set, creating a 98% confidence interval. For the given data the range is 690.0774 lb_f/in to 700.6976 lb_f/in. One is 98% confident that the mean value of the spring constant falls within that data range. For the experimental data set: one is 98% confident that the mean spring constant value falls between 697.7362 lb_f/in to 710.7598 lb_f/in. Due to the interval being calculated at 98% confidence, the probability that the mean will not fall within that data range is 0.01 in either side of the confidence interval.

The comparative analysis compares both the experimental and given data set. The result of the comparative analysis determines whether or not there is variance between the two data sets. The null hypothesis for the comparative test states that there is no variance between the two data sets. The comparison is between a calculated t value of 3.354. The calculated t_0 value was compared to a table value found by using the parameters of a 98% confidence interval. The t value is; 2.718. Since these values are not equal, the null hypothesis is rejected. There is variance between the two data sets. A suggestion for repeating this analysis would be to increase the confidence interval. With a higher confidence interval, the area under the curve corresponding to the null hypothesis would not invade the shaded area under the curve corresponding to the α level.

7 Conclusions and Recommendations

Statistical analyses are used in research to interpret and communicate research findings. These finding may or may not support the hypotheses assumed prior to conducting experiments. By performing statistical analysis, results can also support the methodology and conclusions. All statistical analyses are performed at 98% confidence. The experimental and given data sets underwent comparative analysis. Comparative analysis dictates the variance between the two data sets. The initial null hypothesis assumed true is that there is no variance between the data sets. Specifically, this refers to the spring constant value and its deviation which can be represented by the t values. The t_0 value is calculated using statistical observations from the data sets and the other recorded from the t distribution table based on the sample size and α level. The calculated value of t_0 is 3.354, and the t value from the table is 2.718. Because the values are not equal, the null hypothesis is rejected. This means that there is variance between the two data sets. As a result, if a comparative analysis were to be repeated of these two data sets, a larger confidence interval would be suggested.

Confidence intervals at 98% of both the given and experimental data sets were calculated and plotted, which can be seen in figures 3 and 4. This confidence interval is associated with an alpha level of 0.01. The α level signifies the probability of rejecting the null hypothesis. The null hypothesis in this case is that the mean of the data set will fall within the data range specified by the confidence interval. For the given data, the 98% confident states that the mean of the give spring constants will fall within 690.0774 lb_f/in and 700.6976 lb_f/in. For the experimental data, the 98% confident states that the mean of the experimental spring constants will fall within 697.7362 lb_f/in and 710.7598 lb_f/in.The null hypothesis is accepted for both data sets.

An outlier analysis was performed on the data sets using Thompson's τ Test. This test creates a threshold value on the amount of deviation a data value can have from the mean, using equation (7). The τ value is determined by the sample size. The deviation of a data point from the mean is determined using equation (4). If the deviation of a data point from the mean is larger than the threshold value deviation, then that value is considered an outlier and is rejected. The Thompson's τ Test was repeated until the data set no longer contained outliers. For the given data, the test had to be iterated three times until the data set was finalized. For the given data the mean used for all analyses is 695.3875 lb_f/in and the standard deviation is 5.0098 lb_f/in. For the experimental data, the Thompson's τ Test only had to be performed once, as there were no outliers in the set. The mean spring constant for the experimental data is 704.248 lb_f/in with a standard deviation of $\pm$3.8886 lb_f/in.

If this statistical analysis were to be repeated, some changes would be made. The comparative analysis results reject the null hypothesis which means that there is variance between the two data sets. Due to this result, it may be a beneficial to increase the confidence interval. The result of this comparative analysis test implies that the area under the curve that correspond to the null hypothesis lies well within the shaded area under the curve, corresponding to the α value. This indicates that a smaller α value can be used in the analysis of this data. To improve result and accuracy more trials should be run.

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9 Appendices

Table 8: Experimental Coiled Spring Data

Coiled Spring	Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
Target Depth (in)	Actual Depth (in)	Load (lbf)	Actual Depth (in)	Load (lbf)	Actual Depth (in)	Load (lbf)	Actual Depth (in)	Load (lbf)	Actual Depth (in)	Load (lbf)
0.05	0.0502	19.2900	0.0500	22.0500	0.0502	15.9900	0.0500	22.0500	0.0502	22.1300
0.10	0.1000	48.1100	0.1000	50.5500	0.1000	42.3300	0.1000	50.5500	0.1002	50.9700
0.15	0.1502	80.6500	0.1500	82.5500	0.1502	73.6800	0.1500	82.5500	0.1498	82.9600
0.20	0.2001	113.8000	0.2005	115.9000	0.2003	106.7000	0.2005	115.9000	0.1999	116.0000
0.25	0.2503	147.7000	0.2505	149.6000	0.2501	140.1000	0.2505	149.6000	0.2507	150.4000
0.30	0.2999	182.3000	0.3001	184.5000	0.3003	174.6000	0.3001	184.5000	0.3008	185.6000
0.35	0.3500	219.6600	0.3503	222.0000	0.3500	211.6000	0.3502	222.2000	0.3506	223.0000
0.40	0.4006	257.8000	0.4002	260.3000	0.4000	249.7000	0.4001	260.3000	0.4001	260.6000
0.45	0.4501	295.6000	0.4500	298.6000	0.4501	288.1000	0.4500	298.6000	0.4503	299.1000
0.50	0.5000	332.4000	0.5003	336.7000	0.5003	326.9000	0.5001	336.7000	0.5000	336.2000
	Correct Depth (in)	Correct Load (lbf)	Correct Depth (in)	Correct Load (lbf)	Correct Depth (in)	Correct Load (lbf)	Correct Depth (in)	Correct Load (lbf)	Correct Depth (in)	
0.0000	0	0.0000	0	0	0	0	0	0	0	0
0.0502	0.0163	19.2900	0.0200	22.0500	0.0056	15.9900	0.0200	22.0500	0.0209	22.1300
0.1000	0.0661	48.1100	0.0700	50.5500	0.0554	42.3300	0.0700	50.5500	0.0709	50.9700
0.1502	0.1163	80.6500	0.1200	82.5500	0.1056	73.6800	0.1200	82.5500	0.1205	82.9600
0.2001	0.1662	113.8000	0.1705	115.9000	0.1557	106.7000	0.1705	115.9000	0.1706	116.0000
0.2503	0.2164	147.7000	0.2205	149.6000	0.2055	140.1000	0.2205	149.6000	0.2214	150.4000
0.2999	0.2660	182.3000	0.2701	184.5000	0.2557	174.6000	0.2701	184.5000	0.2715	185.6000
0.3500	0.3161	219.6600	0.3203	222.0000	0.3054	211.6000	0.3202	222.2000	0.3213	223.0000
0.4006	0.3667	257.8000	0.3702	260.3000	0.3554	249.7000	0.3701	260.3000	0.3708	260.6000
0.4501	0.4162	295.6000	0.4200	298.6000	0.4055	288.1000	0.4200	298.6000	0.4210	299.1000
0.5000	0.4661	332.4000	0.4703	336.7000	0.4557	326.9000	0.4701	336.7000	0.4707	336.2000
0.5500	0.5161	364	0.5199	364	0.5054	364	0.5199	364	0.5206	364
0.6000	0.5661	400	0.5699	400	0.5554	400	0.5699	400	0.5706	400

Table 9: Thompson's Table

n	τ	n	τ	n	τ
3	1.1511	21	1.8891	40	1.9240
4	1.4250	22	1.8926	42	1.9257
5	1.5712	23	1.8957	44	1.9273
6	1.6563	24	1.8985	46	1.9288
7	1.7110	25	1.9011	48	1.9301
8	1.7491	26	1.9035	50	1.9314
9	1.7770	27	1.9057	55	1.9340
10	1.7984	28	1.9078	60	1.9362
11	1.8153	29	1.9096	65	1.9381
12	1.8290	30	1.9114	70	1.9397
13	1.8403	31	1.9130	80	1.9423
14	1.8498	32	1.9146	90	1.9443
15	1.8579	33	1.9160	100	1.9459
16	1.8649	34	1.9174	200	1.9530
17	1.8710	35	1.9186	500	1.9572
18	1.8764	36	1.9198	1000	1.9586
19	1.8811	37	1.9209	5000	1.9597
20	1.8853	38	1.9220	to->∞	1.9600

Table 10: t-distribution table

	$\alpha/2$						
DOF	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
∞	1.282	1.645	1.960	2.326	2.576	3.091	3.291
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.920	4.303	6.965	9.925	22.328	31.600
3	1.638	2.353	3.182	4.541	5.841	10.214	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.660
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
1000	1.282	1.646	1.962	2.330	2.581	3.098	3.300