Glide-Master 3000

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MCE 454: Tribology
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Information Page

a. Slider Name: Glide-Master 3000
b. Lubricant: SAE 10 Oil
c. Mass of Slider: 500 grams
d. Slider Material: Aluminum 6061-T6
e. Distance Prediction: 20.41 inches

Problem Statement

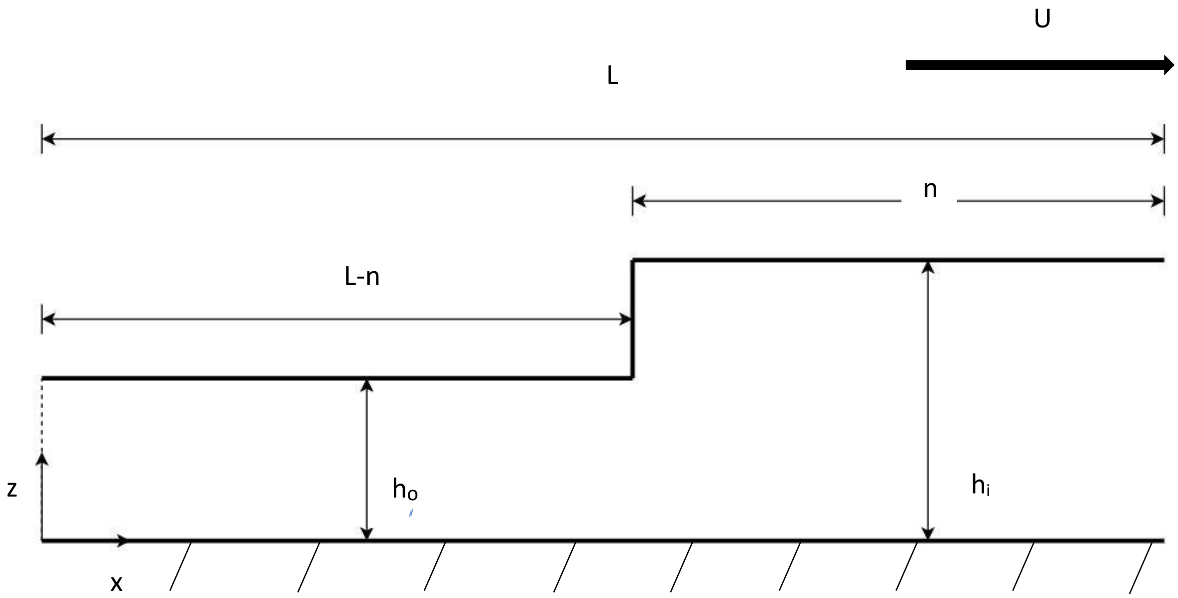
The objective of this design project is to create an optimized slider bearing which, with the aid of lubricant, slides the longest distance possible on a given surface after impact with a rubber ball attached to a pendulum.

Constraints for the slider bearing itself, track surface and pendulum-ball system were provided. The slider bearing must have a mass greater than or equal to 500 g, a width less than or equal to 10 cm and a length less than or equal to 10 cm. There were three different material options for the composition of the slider bearing: nylon 6,6, cold rolled steel 1018 and aluminum 6061-T6. The only lubricant restriction was that it must be a liquid and approved by the professor. The track surface is glass with a width including side rails of 30.5 cm, a length of 2.4 m and a negligible angle of inclination. The pendulum system is composed of a pendulum arm that has a length of 0.36 m and a mass of 61.6 g. On the end of the pendulum arm is a rubber ball with a diameter of 2.473 in and a mass of 143.9 g. A connecter piece with a mass of 52.3 g serves as the hinge allowing the pendulum to swing. This system will be released at an angle of 45 degrees from the resting position.

Nomenclature

B……Width of Slider Bearing (in)
d……Distance Slid (in)
E_f……Frictional Energy (ft*lbf)
F_n……Normal Force (lbf)
F_f……Friction Force (lbf)
h……Height of Slider Bearing (in)
h_o……Outlet Height (in)
h_i……Inlet Height (in)
h_p……Height of Ball on Pendulum at Strike Point (in)
L……Length of Slider Bearing (in)
m_p……Total Mass of Pendulum System (slug)
m_s……Mass of Slider Bearing (slug)
n……Length of Stepped Section (in)
P……Pressure (lbf/in^2)
Q……Volumetric Flow Rate (ft^3/s)
S_h……Step Height (in)
t……Time (s)
u……Velocity of Lubricant (ft/s)
U……Velocity of Slider Bearing (ft/s)
V……Volume of Slider Bearing (in^3)
W……Load (lbf)
x,y,z……Dimensional Coordinates
ρ……Lubricant Density (lbm/ft^3)
η……Dynamic Viscosity (lbf*s/in^2)

Figure 1



Slider Bearing Design Solution and Calculations

i)

The material for this slider bearing will be aluminum 6061-T6 which has a density of 0.0975 lb/in³ (Ref. 1). This material was chosen due to it possessing the lowest average coefficient of friction of the three material choices. Aluminum is also an easier material to mill and will yield a superior surface finish than the other two materials (Ref. 2).

ii)

Using the x-direction Navier-Stokes equation:

$$\rho\left(\frac{\delta u}{\delta t}+u\frac{\delta u}{\delta x}+w\frac{\delta u}{\delta z}\right)=-\frac{\delta p}{\delta x}+\eta\frac{\delta^2 u}{\delta z^2}\ \text{(Ref. 3)}$$

Steady, fully developed flow and a negligible flow in the *z* direction can be assumed. The x-direction Navier-stokes can then be reduced to the equation:

$$\frac{\delta^2 u}{\delta z^2}=\frac{1}{\eta}\frac{\delta p}{\delta x}$$

Integrating for the fluid velocity in the x-direction, *u*, once with respect to *z* yields the equation:

$$\frac{\delta u}{\delta z}=\left(\frac{1}{\eta}\frac{\delta p}{\delta x}\right)z+c_1$$

Integrating again for the fluid velocity *u* with respect to *z* yields the fluid velocity profile in the x-direction:

$$u(z)=\left(\frac{1}{\eta}\frac{\delta p}{\delta x}\right)\frac{z^2}{2}+c_1 z+c_2$$

Referring to Figure 1, the boundary conditions for this problem are:

$$@\ z=0:u=0\quad(1)$$

$$@\ z=h:u=U\quad(2)$$

Substituting boundary condition (1) into the fluid velocity profile:

$$c_2=0$$

Substituting boundary condition (2) and the value of c_2 into the fluid velocity profile:

$$c_1=\frac{U}{h}-\left(\frac{h}{2\eta}\frac{\delta p}{\delta x}\right)$$

Substituting the values of c_1 and c_2 into the velocity profile and simplifying yields the final x-direction fluid velocity profile:

$$u(z) = \left(\frac{1}{2\eta}\frac{\delta p}{\delta x}\right)(z^2 - zh) + \frac{Uz}{h}$$

The volumetric flowrate of a liquid, Q, is represented by the equation:

$$Q = vA$$

Where v is the velocity of the liquid and A is the liquid's cross-sectional area.

For flow continuity of the lubricant, $Q_{in} = Q_{out} = Q$. Additionally, for flow continuity of the lubricant:

$$\frac{\delta u}{\delta x} + \frac{\delta w}{\delta z} = 0 \quad \text{(Ref. 3)}$$

Where w is the velocity of the lubricant in the z-direction. Integrating this equation with respect to z from 0 to h and taking out a $\frac{\delta}{\delta x}$ from the first term yields:

$$\frac{\delta}{\delta x}\int_0^h u \, \delta z + \int_0^h \frac{\delta w}{\delta z} \, \delta z = 0$$

The volumetric flowrate in the x-direction, q_x, can be determined by the following integration of u, found in the equation above:

$$q_x = \int_0^h u \;\; \delta z \quad \text{(Ref. 3)}$$

Substituting in for the velocity of the lubricant in the x-direction, u:

$$q_x = \int_0^h \left(\frac{1}{2\eta}\frac{\delta p}{\delta x}\right)(z^2 - zh) + \frac{Uz}{h} \; \delta z$$

Integrating with respect to z from 0 to h and simplifying yields the x-direction volumetric flow rate equation:

$$q_x = -\left(\frac{h^3}{12\eta}\frac{\delta p}{\delta x}\right) + \frac{Uh}{2}$$

Knowing:

$$-(n)\frac{\delta p}{\delta x} = p_{\max(i)} \text{ and } (L-n)\frac{\delta p}{\delta x} = p_{\max(o)} \quad \text{(Ref. 3)}$$

And $p_{max(i)} = p_{max(o)} = p_{max}$, therefore:

$$\left(\frac{\delta p}{\delta x}\right)_i = -\frac{p_{max}}{n} \text{ and } \left(\frac{\delta p}{\delta x}\right)_o = \frac{p_{max}}{(L-n)}$$

If $Q_{in} = Q_{out}$, then $q_{x(i)} = q_{x(o)}$. Substituting the relations above into the flow continuity of the lubricant relation yields:

$$-\frac{{h_i}^3}{12\eta}\left(\frac{\delta p}{\delta x}\right)_i + \frac{Uh_i}{2} = -\frac{{h_o}^3}{12\eta}\left(\frac{\delta p}{\delta x}\right)_o + \frac{Uh_o}{2}$$

$$-\frac{{h_i}^3}{12\eta}(-\frac{p_{max}}{n}) + \frac{Uh_i}{2} = -\frac{{h_o}^3}{12\eta}(\frac{p_{max}}{L-n}) + \frac{Uh_o}{2}$$

Solving for p_{max} and simplifying yields the final p_{max} equation:

$$p_{max} = \frac{6U\eta(h_o - h_i)}{\frac{{h_i}^3}{n} + \frac{{h_o}^3}{L-n}}$$

The derivation for p_{max} and the x-direction velocity profile *u* have been modified for this final design report. Due to a difference in boundary conditions from the original report, the derivations above have been altered to reflect these changes.

Taking the dimensionless constants:

$$H = \frac{h_i}{h_o} \text{ and } G = \frac{n}{(L-n)} \text{ (Ref. 3)}$$

To determine load, *W*, per finite width of the slider bearing into the board, *B*, substitute the above ratios into the simplified p_{max} equation:

$$p_{max} = \frac{G(H+1)}{H^3+G}\frac{6U\eta(L-n)}{{h_0}^2}$$

If $W = p*A$ where $A = \delta y \delta x = B$ then:

$$\frac{W}{B} = \int_0^1 p\ \delta x = \frac{1}{2}p_{max}(n(L-n)) \text{ (Ref. 3)}$$

The following formula is given after substitution and simplifying:

$$\frac{W}{B} = \left(\frac{U\eta L^2}{{h_o}^2}\right)\left[\frac{3G(H-1)}{(1+G)(H^3+G)}\right] = \frac{Wh_o^2}{BU\eta L^2}$$

If:

$$\frac{W}{B} = W^*$$

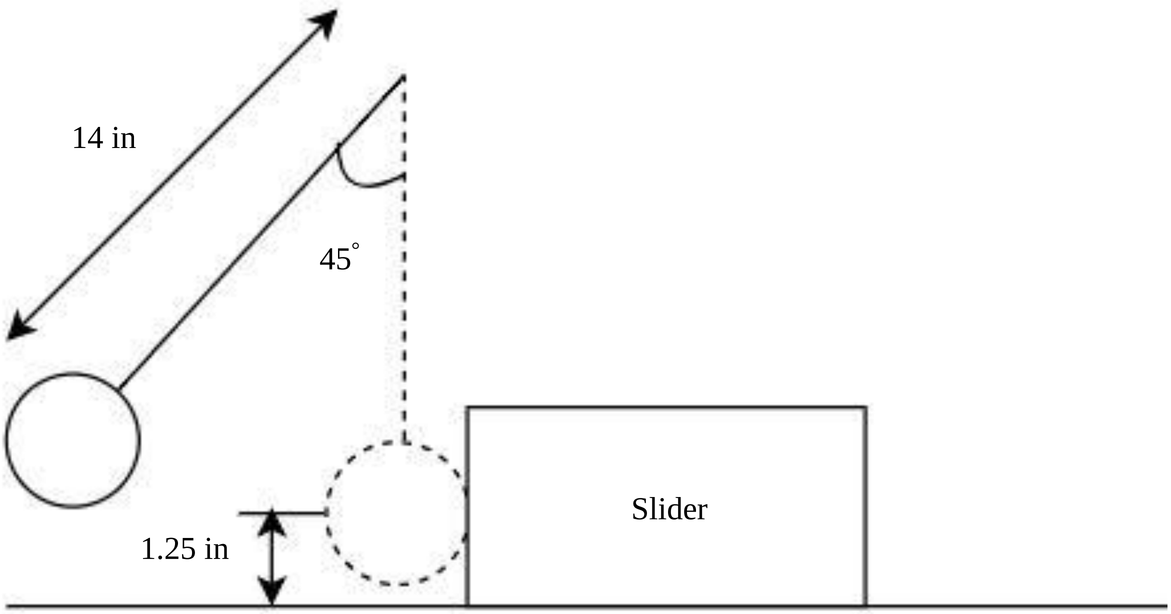
Then the constant W^* for optimization of load carrying capacity:

$$W^* = \frac{Wh_o^2}{BU\eta L^2} = 0.206 \text{ (Ref. 3)}$$

One can solve for the ideal dimensionless ratios of:

$$H = 1.87 \text{ and } G = 2.59 \text{ (Ref. 3)}$$

Figure 2



iii)

Given the derivations above, the ideal slider bearing can now be made. First, the equation for G was set up:

$$\frac{n}{L-n} = 2.59$$

In a long bearing approximation, it is assumed that the length is much bigger then the width. To accurately reflect this, the maximum length allowed for L, 3.937 in, was used.

By plugging *L* into the above equation, it was determined that *n* = 2.840 in. This means *L* – *n* = 1.10 in. When the pendulum ball is perpendicular to the surface, the distance from the center of the ball to the surface is 1.25 in. The height of the bearing, *h*, was then decided to be 2.5 in to line up the center of mass of the ball with the center of mass of the bearing. The mass of the bearing was determined to be 500 g (0.0343 slug) to keep the bearing as light as possible. The following equation for mass was used to determine the bearing width, *B*:

$$m = \rho V$$

Where the equation for the volume of the bearing is:

$$V = LBh$$

Knowing the density, length and height, the width *B* was then calculated to be 1.147 in. The next step was to determine h_i and h_o. To do this, the velocity of the slider, *U*, first needed to be determined.

It is assumed energy is conserved from when the ball is not in motion at the start of its swing to when the ball is at the end of its swing. This is represented by the equation:

$$PE = KE \quad \text{(Ref. 4)}$$

Where the equation for the potential energy of the system is:

$$PE_{pendulum} = m_p g h_p \quad \text{(Ref. 4)}$$

m_p represents the combined mass of the pendulum arm, connector piece and the rubber ball which equates to 257.8 g (0.0177 slug). Referring to Figure 2, the height of the ball at the end of the pendulum system from the striking point, h_p, can be determined by:

$$\cos(45) = \frac{h_p}{14\ in}$$

h_p is then calculated to be 9.9 in. Therefore, the potential energy of the system can be calculated to be 15.088 ft*lbf using the equation above.

If energy is also conserved in the collision between the pendulum and the slider, the collision is assumed to be perfectly elastic, and the ball at the bottom of the pendulum does not bounce after the impact between the pendulum and the slider, the following equations are considered. With *i* representing the energy prior to the collision and *f* representing the energy after the collision:

$$KE_{pendulum_i} + KE_{slider_i} = KE_{pendulum_f} + KE_{slider_f} \quad \text{(Ref. 4)}$$

Knowing the slider is not in motion before the collision and that the ball is not in motion after the collision, the above equation can be reduced to:

$$KE_{pendulum_i} = KE_{slider_f}$$

Knowing that the kinetic energy of the pendulum system at the bottom of the swing, or the contact point, is equal to the potential energy of the pendulum system at the release point, one can reduce the two equations above to obtain the equation:

$$PE_{pendulum} = KE_{slider}$$

Where:

$$KE_{slider} = \frac{1}{2} m_s U^2$$

Using the conservation of energy equation above, the velocity of the slider, *U*, was determined to be 5.234 ft/s.

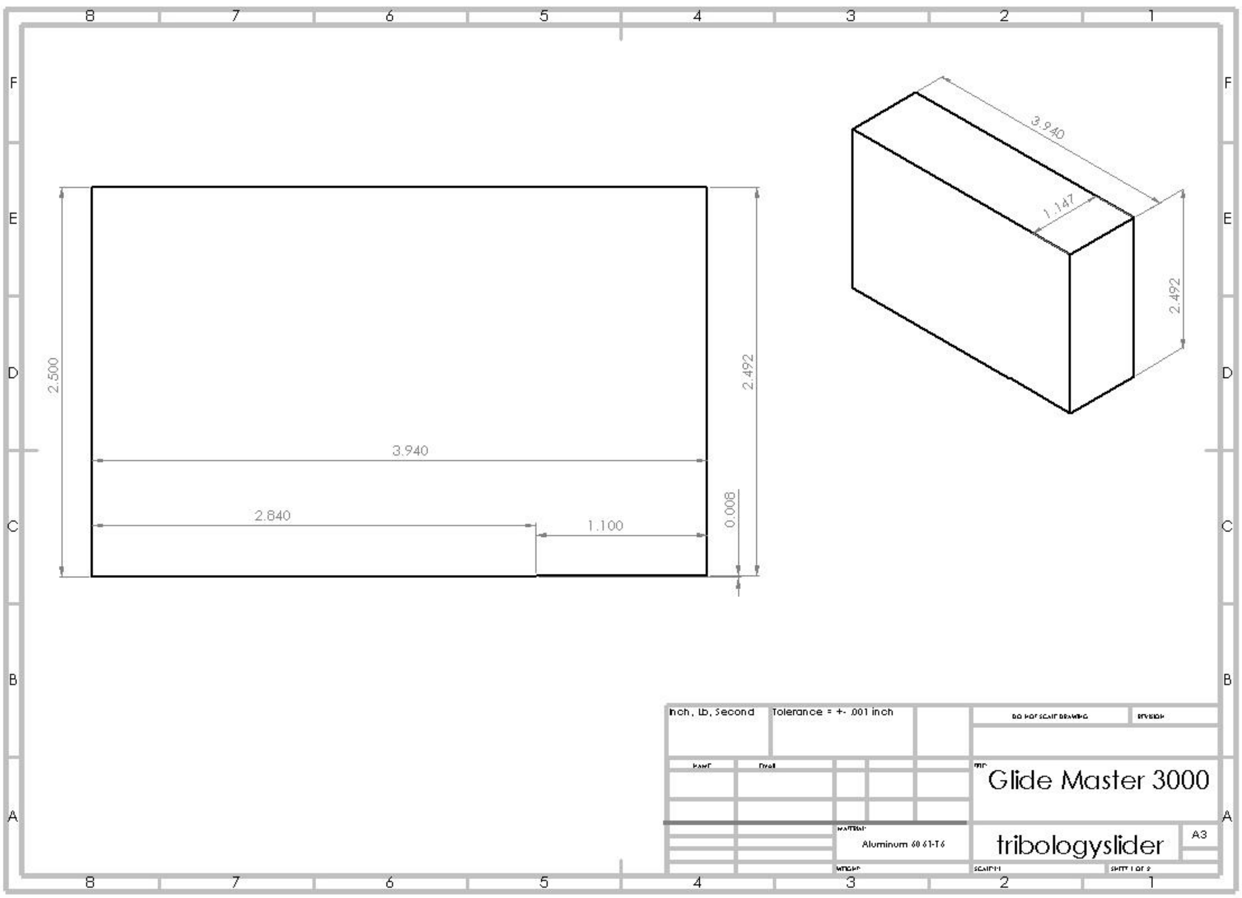
The last variable needed was the dynamic viscosity, η, which was determined from the lubricant chosen for the surface. The lubricant chosen was SAE 10 oil which has a dynamic viscosity of 1.64×10^{-3} $lbf*s/ft^2$ (Ref. 5). These variables were plugged into the W^* equation found in the derivation above which calculated h_o to be 0.015 in. The inlet height, h_i, was then determined to be 0.028 in by substituting h_o into the ratio for dimensionless *H* found above. Therefore, the step height, S_h, was determined to be 0.013 in.

The values for h_i, h_o, S_h and *U* have been changed from the original design report. After further review, it was decided to approach the kinematic analysis differently, leading to modifications in the values for inlet, outlet and step heights as well as the velocity of the slider bearing.

iv)

Using information from part two, a three-dimensional geometry was designed. The attached drawing on the following page reflects this model.

Old Drawing



3.940
1.147
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2.492
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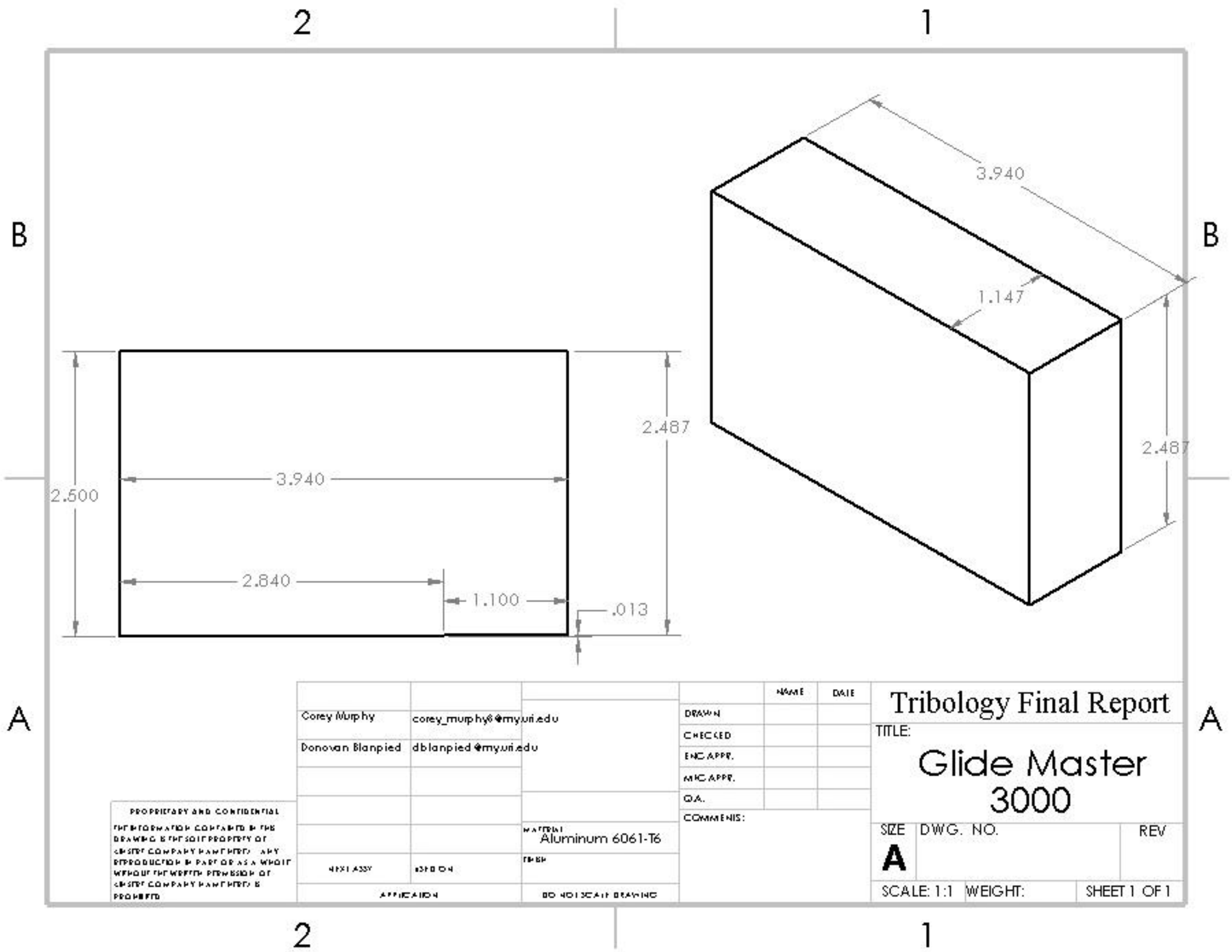
Inch, Lb, Second
Tolerance = +- .001 inch
Glide Master 3000
Aluminum 6061-T6
tribologyslider
A3

New Drawing



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Corey Murphy
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Donovan Blanpied
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SHEET 1 OF 1

Performance Prediction

To find the total distance slid by the slider bearing, kinetic energy had to be considered. By the time the slider has come to rest, all the initial kinetic energy will have been converted into frictional energy, which is equal to 15.088 ft*lbf. Frictional energy is equal to the frictional force multiplied by the distance slid:

$$E_f = F_f * d$$

Where:

$$F_f = F_n * \mu$$

μ is the coefficient of friction of metal on glass which was determined to be 0.25 (Ref. 5). F_n represents the normal force of the glass on the bearing which was determined to be 35.484 lbf by multiplying the weight of the bearing (1.102 lbs) by the gravitational constant of 32.2 ft/s^2. F_f was then calculated to be 8.871 lbf.

By plugging these numbers into the formula for frictional energy, the distance, *d*, can be determined. Therefore, the performance estimation for the distance the slider bearing will slide based on this calculation is 20.41 in.

Conclusions

After completing the necessary steps to optimize a Rayleigh-Step Slider Bearing, a final performance prediction for the distance the bearing will slide after impact with a rubber ball and pendulum system was determined. Based on these optimized calculations, this prediction theoretically should be the maximum distance this slider bearing will travel with the chosen lubricant.

The velocity profile for the lubricant, $u(z)$, was first determined by utilizing the x-direction Navier-Stokes equation. This equation was reduced based on given initial conditions and then integrated twice. Boundary conditions relevant to this slider bearing were used to solve for the two constants, yielding a final velocity profile for the fluid lubricant.

An x-direction volumetric flowrate, q_x, for the fluid lubricant was determined by integrating the velocity profile with respect to z from 0 to h. The flow continuity relation of $Q_{in} = Q_{\text{out}}$ was then used in conjunction with the max inlet and outlet pressure ($p_{max(i)}$ and $p_{max(o)}$) relations to determine a final, simplified equation for the maximum pressure, p_{max}. This equation was then used to derive the values of the dimensionless ratios G and H.

The length of the bearing, L (3.937 in), and the length of bearing's step, n (2.840 in), was determined using the long-bearing approximation and the dimensionless ratio G. Utilizing simple mass and volume equations for the slider bearing, the width, B (1.147 in), was calculated.

Once these values were determined, a kinematic analysis could be performed on the pendulum system striking the slider bearing. Assuming energy is conserved when the ball is not in motion at the beginning of its swing to when the ball is at the end of its swing, the conservation of energy equation was used. This allowed for the potential energy of the pendulum system to be equated to the kinetic energy of the slider bearing. The calculated value for the potential energy of the pendulum system (15.088 ft*lbf) was then equated to the kinetic energy of the slider to determine the velocity of the slider bearing, U (5.234 ft/s).

The value for the outlet height, h_o (0.015 in), was then calculated substituting the necessary values into the given W^* equation. The inlet height, h_i (0.028 in), was then solved using the dimensionless ratio H. With both the inlet and outlet heights determined, the step height, S_h (0.013 in), was found.

A final performance prediction was calculated using equations for friction force, F_f (8.871 lbf), and friction energy, E_f (15.008 ft*lbf). Substituting the necessary values into the friction energy equation, the estimated distance the slider bearing should travel, d, was determined to be 20.41 in.

Theoretically, if the conditions used throughout this analysis are replicated on the day of the slider bearing trial, the maximum distance this optimized Rayleigh-Step Slider Bearing will travel with the chosen fluid lubricant is 20.41 in.

References

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